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Detecting random walk in stock market prices based on Markov chains: Examining The Mexican Stock Market Index

Detección de caminata aleatoria en precios bursátiles mediante cadenas de Markov: aplicación al Índice de Precios y Cotizaciones de México

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ABSTRACT

The Random Walk (weak form efficient market) hypothesis is of vital importance in economics and finance to explain the behaviour of asset prices. Several authors have examined the validity and conditions under which the hypothesis holds. Most of the techniques and models used, rely on runs and serial correlation tests, however test using Markov chains are rare. Most Markov chains applications perform an stratification of returns defining the structure of the state space. The aim of this research is to detect the presence of random walk in stock market returns using Markov chains. The chain states are defined as the run lengths the process can develop. The concept of cycles is also introduced modelling the process in a more concretely. Conclusions are drawn analysing stationarity of the steady state probability distributions under diverse scenarios. The Mexican stock market daily closing prices index is analysed, covering a 16-year period, finding that the random walk is not present. This result is corroborated applying conventional random walk hypothesis tests.

JEL Classification: C02, C65, G14

Keywords: *stock returns, random walk, Markov chains, runs, cycles, steady state.*

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Resumen

La hipótesis de caminata aleatoria (forma débil de mercado eficiente) es de vital importancia en economía y finanzas para explicar el comportamiento de precios bursátiles. Una gran cantidad de artículos han examinado la validez y condiciones bajo las cuales se cumple la hipótesis. La gran mayoría de las técnicas y modelos que se han aplicado para confirmar la hipótesis se han basado en pruebas de corridas y de correlación serial, siendo raro encontrar la aplicación de Cadenas de Markov. En la mayoría de las aplicaciones de Cadenas de Markov se ha realizado la estratificación del rendimiento para estructurar el espacio de estados de la cadena. El objetivo de esta investigación, es el detectar la existencia de caminata aleatoria en los rendimientos bursátiles, mediante la aplicación de cadenas de Markov. Se definen los estados de la cadena como la longitud de la corrida que el proceso pueda generar. Se introduce además el concepto de ciclos, con el propósito de modelar el proceso de forma más concreta. Se obtienen conclusiones, analizando la estacionariedad en las distribuciones de probabilidad en condiciones de estado estable, observadas en escenarios diversos. Como ejemplo de aplicación de esta técnica de análisis se toma el caso del Índice de Precios y Cotizaciones (IPC) del mercado de valores mexicano, considerado un período de estudio de 16 años. Se concluye la ausencia de caminata aleatoria, y se corrobora este resultado con la aplicación de pruebas de hipótesis convencionales.

Clasificación JEL: C02, C65, G14

Palabras clave: rendimientos bursátiles, caminata aleatoria, cadenas de Markov, corridas, ciclos, estado estable.

Introduction

The random walk is known in stochastic processes theory to have the memoryless property or Markov property. The Markov property is fundamental in time series analysis and its validity has important implications in economics and finance.

McQueen and Thorley (1991) test the random walk hypothesis based on the statistical theory of finite state Markov processes or Markov chains. They perform a Bernoulli discretization (high, low) of a series of annual stock returns taking the average return of the prior 20 years as a reference base. Annual real and excess returns are shown to exhibit significant non-random walk tendencies in the sense that low (high) returns tend to follow runs of high (low) returns.in the post-war period.

The simple time-homogeneous Markov model is one the most popular models specifying the stochastic process by transition probabilities (Jarrow *et al*. 1997). Chen and Hong (2012) have developed a new test for the Markov property using the conditional characteristic function embedded in a frequency domain approach, which tests the implication of the Markov property on conditional moments. Chen and Hong give an excellent literature review about models rooted in Markov processes applied to stock market analysis.

Following this line of inquiry, this paper tests the Markov property using long length as stochastic variable, it also introduces the idea of working with cycles, where a cycle is formed by the sequence of two runs of different signs. Cycles offer a simple indicator that is relatively easy to study. The transition probability matrices of runs and cycles are analysed separately, drawing conclusions from their steady-state probability distributions. This method is used to analyse the Mexican stock market prices index (IPC) covering the time period from February 2002 to January 2018, divided into two parts of equal length, obtaining three sample periods overall, period 1, period 2 and a combined period. All results indicate that the Markov property is not present, these results are corroborated with those obtained using conventional random walk hypothesis tests.

The paper is organized as follows. Section 1 presents important definitions. Section 2 introduces and performs an exploratory data analysis of the Mexican stock price dataset. Markov chain modelling is undertaken in Section 3. In section 4 random walk tests are used to corroborate results. Followed by conclusions.

1. Definitions

1.1. Return, run and cycle

The main concern of this research is to test the hypothesis that successive stock market price changes are independent, by applying the Markov chains technique, focusing on the analysis of runs. A run is a sequence of price changes of the same sign (Fama, (1965)). These price changes and their signs are automatically obtained by the calculation of returns. The standard definition of the continuously compounded return or *log return* is used:

$$r_t = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1} \qquad (1)$$

where P_t is the asset price at time t and $p_t = \ln(P_t)$.

To explain these definitions a simple example is considered. Let the sequence of returns (i.e. changes) be: 1.48, 2.08, -0.04, 0.79, -1.13, -0.45, -0.83, 0.25, 0.58 (Note: it is a common practice for returns to be referred to in terms of percentages, but we omit writing % here). Considering only the signs of the variable, the following sequence is observed: +, +, -. +. -, -. -, +, +. Defining a positive run as a sequence of positive returns, a negative run as a sequence of negative returns and run length as the number of observations in a run. In this example there are three positive runs with lengths 2, 1, 2, and two negative runs with lengths 1, 3.

A *cycle* is a sequence of two runs of different signs, this is a negative run followed by a positive run in sequence. For the example, if we start with a negative run there are cycles with lengths 2, 5. Conversely, if a positive run is considered first there are two cycles with lengths 3, 4. In this paper, the cycles are recorded starting with negative runs, and it is necessary that the initial and final run are well defined. Therefore, in the example there would only be one cycle, since although the second negative run is well defined the following positive run is not, as the next observation could be a positive return or negative return.

1.2. Markov Chains

A stochastic process $X = \{X_n : n \geq 0\}$ on a countable set S is a time-homogeneous Markov Chain if, for any $i,j \in S$ and $n \geq 0$,

$$P(X_{n+1} = j|X_0, \ldots, X_n) = P(X_{n+1} = j|X_n) \quad (2)$$

$$P(X_{n+1} = j|X_n = i) = p_{ij} \quad (3)$$

the p_{ij} is called the *one step transition probability* from state i to state j, these transition probabilities satisfy $\sum_{j\in S} p_{ij} = 1$, $i \in S$, while $P = [p_{ij}]$ is called the *(one step) transition probability matrix* of the chain.

Condition (2), called the *Markov property*, says that, at any time n, the next state X_{n+1} is conditionally independent of the past $X_0,\ldots,X_{n-1}$ given the present state X_n. In other words, the next state is dependent on the past and present only through the present state.

Condition (3) simply says that the transition probabilities do not depend on the time parameter n; the Markov chain is therefore "time-homogeneous". If the transition probabilities were functions of time, the process X_n would be a time-inhomogeneous Markov chain. (Serforzo, 2009).

A time-homogeneous Markov chain is entirely defined by the transition probability matrix and the initial distribution $P(X_0 = x_0)$ of the Markov chain.

A Markov chain $(X_k, k \in S)$ is *stationary* if and only if it is time homogeneous, so that X_n has the same probability distribution for all $n \in T$.

2. Exploratory data analysis

The Mexican stock market index IPC is used to illustrate the application of this technique. The data correspond to the daily closing price observations covering a 16-year period (February 2002 to January 2018) and were obtained from *es-us.finanzas.yahoo.com web-site*. The dataset is partitioned into two periods, each 8 years long: February 2002 to January 2010, February 2010 to January 2018. These time intervals will be referred to as 1st period, 2nd period and the whole dataset as the whole period. The analysis of the two subsamples and the whole sample are performed separately. The objective is to detect if the Markovian property is held on runs and on cycles. Table 1 shows the number of observations of the IPC index in the three periods considered.

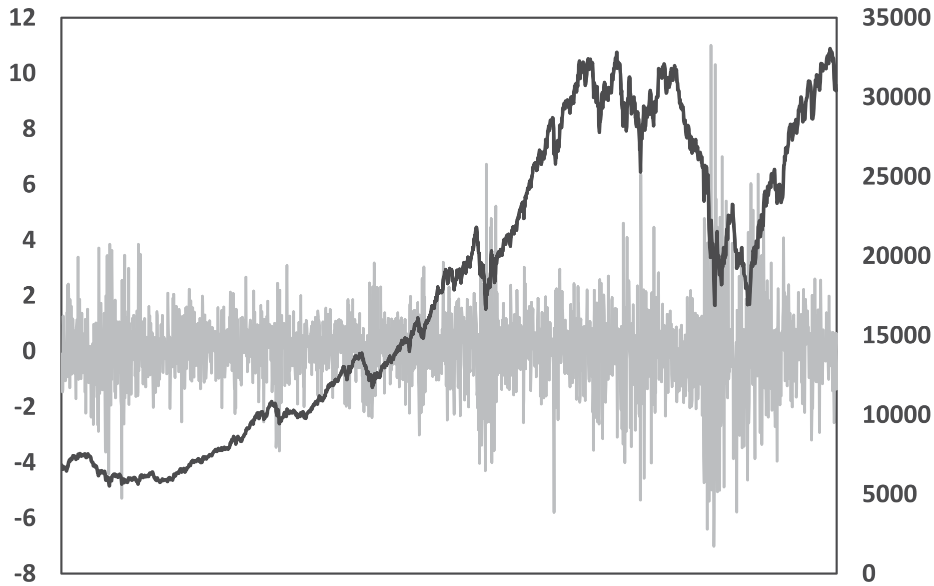
Figure 1 charts the IPC index and returns over period 1, where it grew exponentially until the first week of June 2007, fluctuating around a 30,000 points average for 6 months, and falling dramatically during a short period (until December 27th 2008) losing 48% of its peak value, a recovery followed, going just beyond its previous peak during the last eight months. The

Table 1. Number of observations in IPC index

Sample	Data	Returns (-)	Returns (+)	Returns total
1st period	2011	902	1108	2010
2nd period	1999	951	1047	1998
Whole period	4010	1853	2156	4009

Source: Prepared by author

Figure 1. IPC and returns (%): Feb/2002 – Jan/2010



Source: Prepared by author

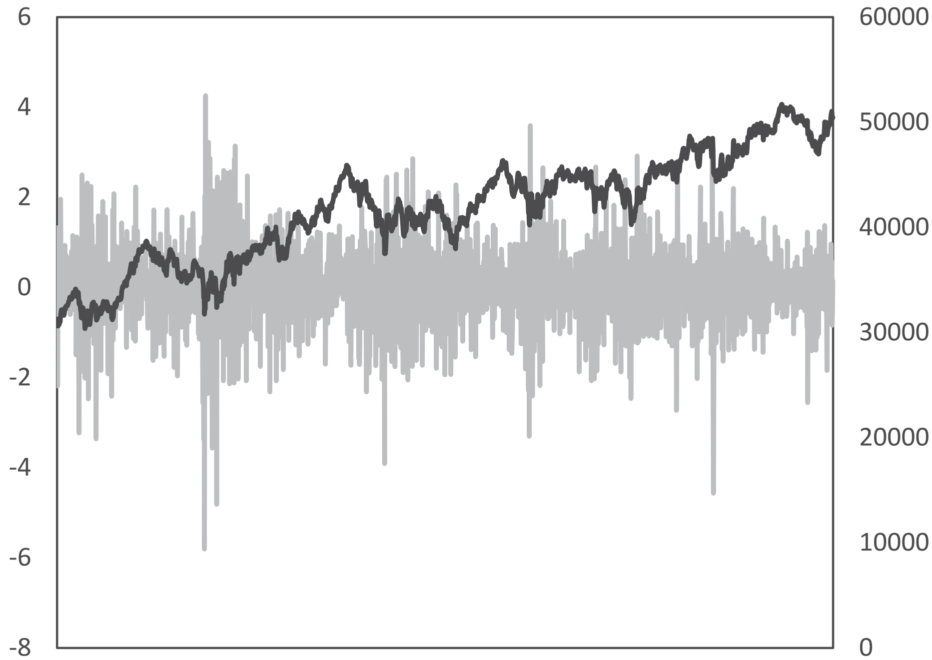
greatest positive return was 10.44% and occurred on December 13th 2008, the greatest negative return was -7.27% and happened on December 22nd 2008.

Figure 2 plots the IPC index and returns over period 2. It shows that movements during period 2 are much more moderate than in period 1, the largest positive return of 4.17% occurred on August 11th 2011 and the largest negative return was -5.98% on August 8th 2011.

Table 2 records the frequencies of positive and negative run lengths for each of the two 8-year periods, and for the combined 16-year period. It can be observed that there are significantly more negative runs of length one than positive runs of length one. It is also observed that negative runs of length lower than 4 occur more frequently than positive runs of the same length, excepting a run of length 3 for the first period. Furthermore, it is evident that positive runs of length longer than 3 present substantially higher occurrence than negative runs of the same length. As expected, the total number of runs is the same for negative runs and for positive runs.

Table 3 records the frequencies of cycle lengths for each of the 8-year periods and for the whole 16-year period. It is observed that in the second period substantially more cycles with length lower than 4 were generated compared to those for the first period, representing 50 per cent of the total

Figure 2. IPC and returns (%): February 2010 – January 2018



Source: Prepared by author

Table 2. Frequency of run length

	Length										
	Run	1	2	3	4	5	6	7	8	9	10	Total
1st period	Negative	234	115	36	22	18	8	4	0	1	-	458
	Positive	188	115	60	37	24	18	6	5	2	3	458
2nd period	Negative	253	119	71	28	13	6	3	0	0	1	494
	Positive	235	117	62	42	22	8	6	1	1	-	494
Whole.	Negative	489	234	127	50	31	14	7	0	1	1	954
Period	Positive	424	233	122	79	46	26	12	6	3	3	954

Source: Prepared by author

number of cycles of the second period. Table 3, also reveals that cycles of length 2 and 3 are the most frequent over the study period, with about 47 per cent of occurrence among the total number of cycles.

Table 3. Frequency of cycle length

	Length												
Sample	2	3	4	5	6	7	8	9	10	11	12	13	Total
1st period	93	105	87	60	41	26	19	13	6	4	3	1	458
2nd period	125	122	88	59	36	30	19	6	7	1	1	-	494
Whole period	219	228	175	119	77	56	38	19	13	5	4	1	954

Source: Prepared by author

3. Markov chains modelling

The aim of modelling a stock market index as a Markov chain is to find out if the market may be viewed as holding the Markov property, i.e., the future is conditionally independent of the past given the present state of the process, and that the probability distribution is time homogeneous.

3.1. Runs

Investigation of the time series of returns focuses on the stochastic variable run length, in contrast to other Markov chains applications where the state space of the chain is defined by stratification of the return´s level see Chen and Hong, (2012).

Let X_n be the length of the run, the time parameter index n indicates when the chain changes sign. The state space is finite $S = \{1, 2, ...,m\}$ representing all possible run lengths, with transition probabilities matrix $P = [p_{ij}]$, where p_{ij} is the probability, that for a run of length *i,* the next run of the same sign be of length *j.* The estimate of the p_{ij} values are simply the transition frequencies from state *i* to state *j* divided by the total number of transitions departing from *i.* Tables 4 to 9 illustrate the transition frequency matrices for these three sample periods.

Table 4. Negative runs transition frequency matrix, first sample period

		Next run length							
		1	2	3	4	5	6	7	9
Previous run length	1	118	67	25	9	8	5	1	1
	2	57	30	16	4	6	1	1	
	3	32	10	6	3	3	0	2	
	4	12	3	3	2	1	1	-	
	5	9	3	3	3	-	-	-	
	6	4	1	2	1	-	-	-	
	7	2	1	1	-	-	-	-	
	9	1	-	-	-	-	-	-	

Source: Prepared by author

Table 5. Positive runs transition frequency matrix, first sample period

		Next run length									
		1	2	3	4	5	6	7	8	9	10
Previous run length	1	74	41	28	14	11	13	2	2	1	2
	2	46	33	14	13	5	2	1	0	1	-
	3	26	15	7	4	3	1	2	1	0	1
	4	18	8	4	2	2	1	0	2	-	-
	5	11	8	2	1	2	-	-	-	-	-
	6	7	4	4	2	1	-	-	-	-	-
	7	3	1	0	0	0	1	1	-	-	-
	8	3	1	0	1	-	-	-	-	-	-
	9	1	1	-	-	-	-	-	-	-	-
	10	0	3	-	-	-	-	-	-	-	-

Source: Prepared by author

Table 6. Negative runs transition frequency matrix, second sample period

		Next run length							
		1	2	3	4	5	6	7	10
Previous run length	1	129	60	40	14	4	4	1	1
	2	57	30	19	6	5	1	1	-
	3	39	18	5	5	2	1	1	-
	4	15	5	4	2	2	-	-	-
	5	7	4	2	-	-	-	-	-
	6	3	1	1	1	-	-	-	-
	7	2	1	-	-	-	-	-	-
	10	1	-	-	-	-	-	-	-

Source: Prepared by author

Table 6 reveals that states 8 and 9 do not occur.

Table 7. Positive runs transition frequency matrix, second sample period

		Next run length								
		1	2	3	4	5	6	7	8	9
Previous run length	1	107	59	30	17	14	5	1	1	1
	2	62	23	16	9	4	2	1	-	-
	3	30	12	8	9	1	1	1	-	-
	4	21	12	3	4	2	-	-	-	-
	5	8	7	1	3	1	0	2	-	-
	6	5	2	0	0	1	-	-	-	-
	7	1	1	3	0	0	0	1	-	-
	8	0	1	-	-	-	-	-	-	-
	9	1	-	-	-	-	-	-	-	-

Source: Prepared by author

Table 8. Negative runs transition frequency matrix, whole sample period

		Next run length								
		1	2	3	4	5	6	7	9	10
Previous run length	1	249	127	65	23	12	9	2	1	1
	2	114	60	35	10	11	2	2	-	-
	3	71	28	11	8	5	1	3	-	-
	4	27	8	7	4	3	1	-	-	-
	5	16	7	5	3	-	-	-	-	-
	6	7	2	3	2	-	-	-	-	-
	7	4	2	1	-	-	-	-	-	-
	9	1	-	-	-	-	-	-	-	-
	10	1	-	-	-	-	-	-	-	-

Source: Prepared by author

Table 9. Positive runs transition frequency matrix, whole sample period

		Next run length									
		1	2	3	4	5	6	7	8	9	10
Previous run length	1	181	101	58	31	25	18	3	3	2	2
	2	108	56	31	22	9	4	2	0	1	-
	3	56	27	15	13	4	2	3	1	0	1
	4	39	20	7	6	4	1	0	2	-	-
	5	19	15	3	4	3	0	2	-	-	-
	6	12	6	4	2	2	-	-	-	-	-
	7	4	2	3	0	0	1	2	-	-	-
	8	3	2	0	1	-	-	-	-	-	-
	9	2	1	-	-	-	-	-	-	-	-
	10	0	3	-	-	-	-	-	-	-	-

Source: Prepared by author

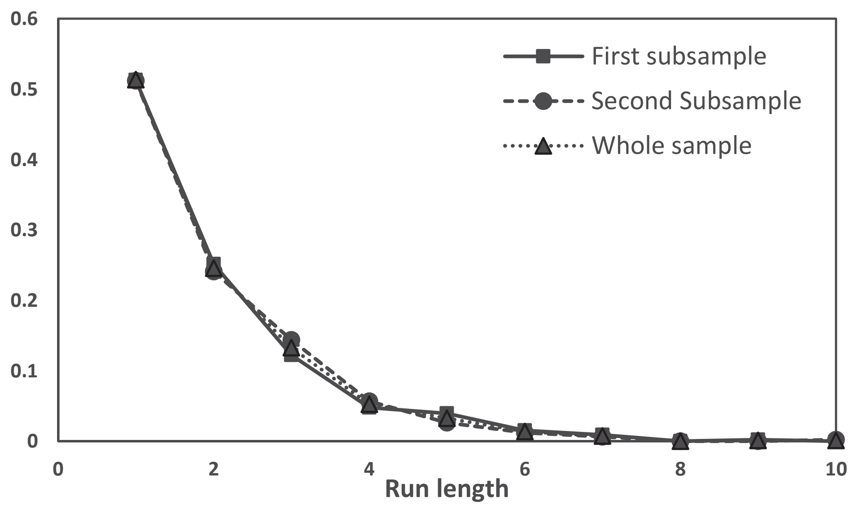
It is assumed that all these chains are aperiodic and irreducible, so that the steady state probability vector π must satisfy

$$\pi = \pi P \tag{4}$$

the π solution is given by any row of the matrix: $\lim_{n \to \infty} P^n$

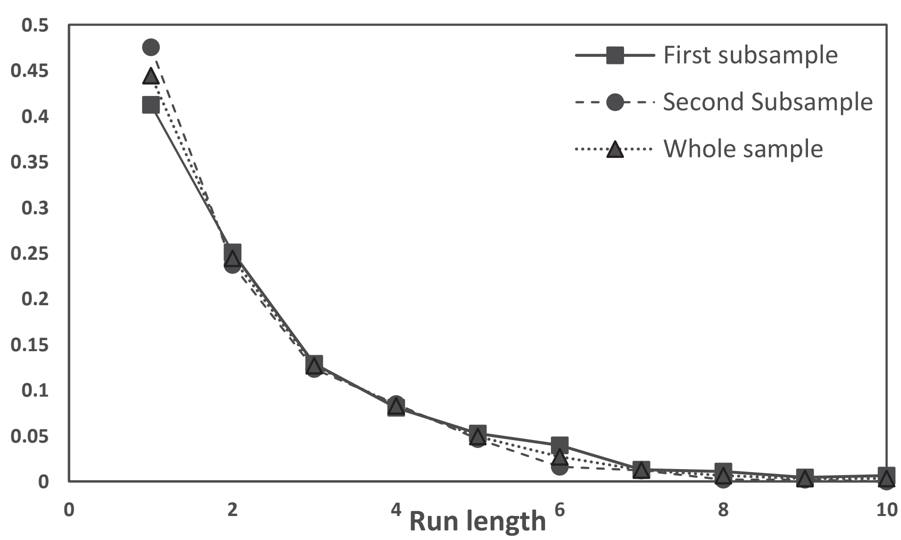
Figures 3 and 4 illustrate the graphs of the steady state probability distributions for positive and negative run lengths for the three sample cases.

Figure 3. Negative run length steady state probabilities



Source: Prepared by author

Figure 4. Positive run length steady state probabilities



Source: Prepared by author

A summary of statistical measures of these steady-state distribution is shown on Table 10.

Table 10. Steady state statistical measures of run length for the sample period

	1st period		2nd period		Whole period	
	Negative	Positive	Negative	Positive	Negative	Positive
Mean	1.9430	2.4064	1.9190	2.1103	1.9286	2.2526
Variance	1.7485	3.1533	1.5886.	2.0848	1.6640	2.6139

Source: Prepared by author

Taking into account that the first and second periods are partition elements of the whole period with same length, an evaluation is carried out about the variation of the second central moment, so that, dividing the negative run length variance of the second period by the negative run length variance of the first period results in a decrement of about 9.14%. A similar evaluation for the positive run length variance results a decrement of about 34%. These results strongly indicate that the steady-state run length distribution does not remain stationary, concluding that the Markovian property is not held on runs.

3.2. Cycles

Let X_n be the length of a cycle, as stated before, a cycle is formed by a positive and negative runs (or vice-versa) in sequence. Here the time parameter index n indicates when the cycle is concluded. The state space is finite S = {2,…,m} representing all possible cycle lengths. As it is evident, length 1 never occurs. The transition probabilities matrix P = [p_{ij}], where p_{ij} is the probability that a cycle of length i be followed by a cycle length j. The estimate of p_{ij} is given by the relative frequency of the transitions from state i to state j. Tables 11 to 14 show these transition frequencies.

Table 11. Cycles transition frequency matrix, first sample period

		Next cycle length											
		2	3	4	5	6	7	8	9	11	12	13	
Previous cycle length	2	14	22	20	15	7	4	7	1	0	2	0	1
	3	16	19	17	14	16	9	2	7	4	0	1	-
	4	16	26	20	9	6	2	6	1	0	1	-	-
	5	21	12	10	7	4	3	1	0	1	1	-	-
	6	6	10	8	5	4	5	0	1	1	0	1	-
	7	8	5	4	2	3	2	2	-	-	-	-	
	8	4	6	3	3	0	0	0	2	0	0	1	-
	9	4	1	4	3	0	0	1	-	-	-	-	-
	10	2	0	0	2	1	1	-	-	-	-	-	-
	11	2	1	1	-	-	-	-	-	-	-	-	-
	12	1	2	-	-	-	-	-	-	-	-	-	-
	13	0	1	-	-	-	-	-	-	-	-	-	-

Source: Prepared by author

Table 12. Cycles transition frequency matrix, second sample period

		Next cycle length										
		2	3	4	5	6	7	8	9	10	11	12
Previous cycle length	2	29	31	20	15	9	10	5	2	3	1	-
	3	37	33	18	15	6	8	3	0	2	-	-
	4	18	19	22	11	5	6	5	1	-	-	-
	5	16	13	10	5	9	3	3	-	-	-	-
	6	7	10	2	7	4	1	2	2	1	-	-
	7	10	7	5	4	2	2	-	-	-	-	-
	8	6	7	3	0	1	0	1	0	1	-	-
	9	1	0	3	0	1	0	0	1	-	-	-
	10	0	2	3	2	-	-	-	-	-	-	-
	11	0	0	1	-	-	-	-	-	-	-	-
	12	1	-	-	-	-	-	-	-	-	-	-

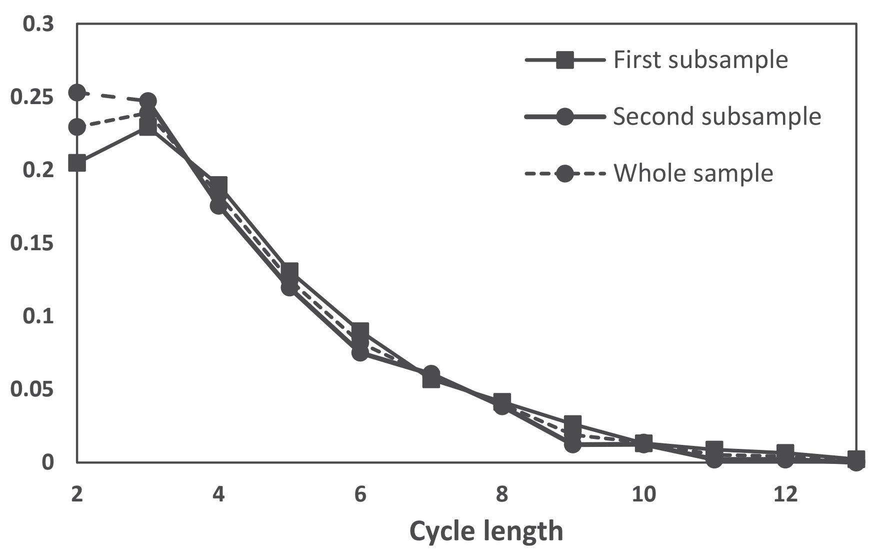
Source: Prepared by author

Table 13. Cycles transition frequency matrix, whole sample period

		Next cycle length											
		2	3	4	5	6	7	8	9	10	11	12	13
Previous cycle length	2	43	54	40	30	16	14	12	3	3	3	0	1
	3	53	52	36	29	22	17	5	7	6	0	1	-
	4	34	45	42	20	11	8	11	2	0	1	1	-
	5	37	25	20	12	13	6	4	0	1	1	-	-
	6	13	20	10	12	8	6	2	3	2	0	1	-
	7	18	12	9	6	5	4	2	-	-	-	-	-
	8	10	13	6	3	1	0	1	2	1	0	1	-
	9	5	1	7	3	1	0	1	1	-	-	-	-
	10	2	2	3	4	1	1	-	-	-	-	-	-
	11	2	1	2	-	-	-	-	-	--	-	-	-
	12	2	2	-	-	-	-	-	--	-	-	-	-
	13	0	1	-	-	-	-	-	--	-	-	-	-

Source: Prepared by author

Figure 5. Cycle length steady state probabilities



Source: Prepared by author

Figure 5 provides a better insight about the behaviour of the cycle length probability distribution at the steady state for the three analysed samples.

A summary of statistical measures for these steady-state distribution is shown in Table 14.

Table 14. Steady state: Statistical measures of cycle length for the sample periods

	1st. Period	2nd period	Whole period
Mean	4.3510	4.0291	4.1818
Variance	4.7469	3.8845	4.3157

Source: Prepared by author

As can be seen on table 14, neither the mean or the variance changed. Therefore, we conclude that the Markovian property does not hold on cycles.

In order to highlight that the cycle length distributions do not preserve time-homogeneity, the two elements of the periods analysed are taken under consideration, since they are the ones suitable for comparison because they have the same length. It is enough to observe the change in variance is about 18% from the first period to the second period, providing strong evidence that the Markov property is not present, consequently, the random walk assumption does not hold.

4. Application of conventional random walk tests

In order to corroborate the previous results the application of runs test and correlation tests will provide evidence about whether or not the random walk hypothesis is fulfilled.

The efficient market hypothesis (EMH) in its weak-form, postulates that successive one-period stock returns are independently and identically distributed (IID), *i.e.*, they resemble a "random walk" (Fama, 1970). Fama (1965) analysed runs for several stocks finding little evidence for violations of efficiency based on serial dependence in returns. Samuelson (1965) and Mandelbrot (1966) rigorously studied the theory of random walks. The EMH has been analysed in many ways, the literature presents a great variety of models to test the hypothesis that markets fluctuations follow a random walk. Examples include: the variance ratio test, the runs test, the serial correlation test and other more general models (for applications of these tests, see for instance Al-Loughani and Chappell, 1997: Chang and Ting, 2000; Sensoy, 2012, Mishra *et al.,* 2012; Risso, 2014, Dsouza and Millikarjunappa, 2015).

Three random conventional tests are applied to the time series under study: difference sign, individual autocorrelation and joint autocorrelation.

4.1. The difference sign test

Kendall (1976) proposed a method to detect randomness by counting the number of positive first differences of the series, which are reflected by returns (see equation 1). Let X represent the number of positive returns of a series having n-1 returns. For a random series the distribution of X tends to be Normal ((n-1)/2, (n+1)/12), see Table 15.

Table 15. The difference sign test result on positive returns,
Ho: Normality holds

Sample period	Positive	Expected	Std-dev.	Confidence Interval (95%)	Decision
1st. period	1108	1004.5	12.95	[979,1029]	Reject Ho
2nd. Period	1047	998.5	12.91	[973,1024]	Reject Ho
Whole period	2156	2004	18.28	[1968,2039]	Reject Ho

Source: Prepared by author

Harvey (1994) supports these findings that the emerging markets returns are not normally distributed.

4.2. Autocorrelation function test: ACF

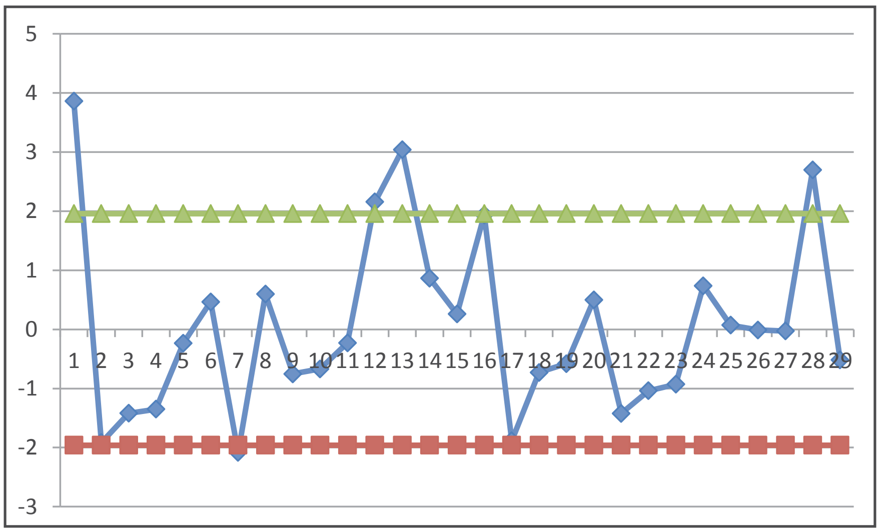
For a given positive integer *l* the t-ratio is statistic defined as

$$t - ratio = p_l / \left(\left(1 + 2 \sum_{i=1}^{l-1} p_i^2 \right) / T \right)^{1/2} \qquad (4)$$

where p_l is the lag-l sample autocorrelation coefficient of r_i it can be used to test $\underline{H}_0$: $p_l = 0$ versus $p_l \neq 0$. If $\{r_t\}$ is a stationary Gaussian series satisfying $p_j = 0$ for $j > l$, the t-ratio is asymptotically distributed as a standard normal random variable. Hence, the decision rule of the tests is to reject H_0 if t-ratio $> Z_{\alpha/2}$, where is the 100(1 - α/2)th percentile of the standard normal distribution (Tsay 2005, p. 27).

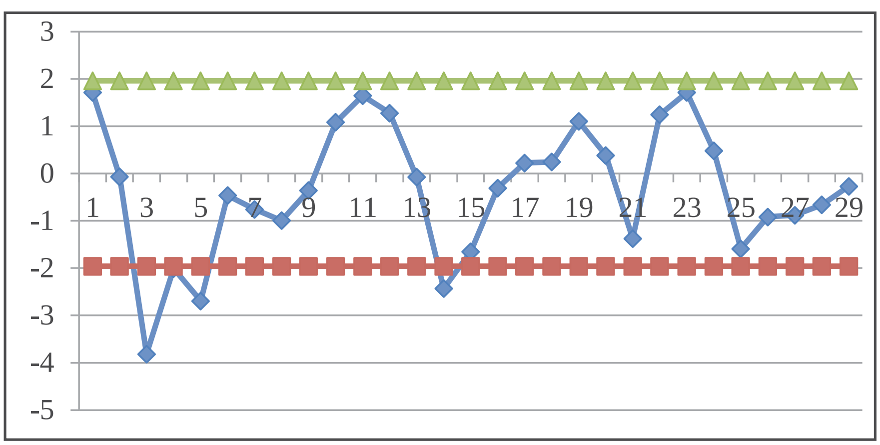
In figures 6 and 7, at least 3 points of the t-ratio statistics fall outside the 95% confidence interval, giving evidence that random walk is not present among the 16-year analysis period.

Figure 6. t-ratio 1st. subsample, 95% confidence interval



Source: Prepared by author

Figure 7. t-ratio, second subsample , 95% confidence interval



Source: Prepared by author

4.3. Ljung and Box test: *Q(m)*

The Ljung and Box statistic *Q(m)* is widely used when it is required to test jointly that several autocorrelations of r_t are zero: $Ho: p_1 = p_2 = \cdot\cdot\cdot = p_m = 0$ against the alternative hypothesis $H_1: p_i = 0$ for some $i \in [\ 1, \ldots m\}$. $\{\ r_t\ \}$ is assumed to be an iid sequence with $E[r_t^2] < \infty$. *Q(m)* is asymptotically a chi-squared random variable with *m* degrees of freedom (Ljung and Box, (1978)):

$$Q(m) = T(T+2)\sum_{l=1}^{m}\frac{p_l^2}{T-l} \quad (5)$$

where p_l is the lag-*l* sample autocorrelation of r_t. Ho is rejected if it is found at least one autocorrelation coefficient is significant. Two additional replicas were performed, *m* = 16, 24, see Table 16.

All these conventional tests provide strong evidence that the null hypothesis of randomness is not held in the IPC index over the study period.

Table 16. Ljung and Box statistic Q(m)

Sample	Q(*m*=8)	Q(*m*=16)	Q(*m*=24)
1st. period	27.66	47.92	57.58
2nd. Period	30.93	45.86	54.35
Whole period	45.67	67.72	77.92
$\chi^2_{m,5\%}$	14.07	26.30	36.42
Decision	Reject Ho	Reject Ho	Reject Ho

Source: Prepared by author

The results obtained with the application of these three methods testing randomness, build upon the first difference of the IPC time series and provide strong evidence that the random walk hypothesis is not present in the time series during the study period. The difference sign test focussing on the number of positive returns and assuming normality (Ho), rejects Ho, since the results fail within the 95% confidence interval. The other two methods used are based on individually and jointly autocorrelation tests and also provide evidence that the random walk hypothesis does not hold for the IPC time series at 5% significance level.

Conclusions

In this paper a new approach is introduced for the search of randomness in stock market returns. This approach involves the application of Markov chains using run length as the stochastic variable. In this analysis the concept of *cycle* is also introduced, which consists of two runs of different signs in sequence. The main objective is to detect if the Markov property holds for a series of returns. The analysis is carried out using the Mexican Stock Market Index for a 16-year period of daily stock closing prices. A division of the dataset is done obtaining two periods, each 8 years long. Dealing with the three sample periods as separate cases, we determine the stochastic matrices with state spaces consisting of the possible lengths of runs and cycles in the three periods. By examining the second central moment of the steady-state probability distributions, conclusions are drawn about homogeneity and stationarity properties of the series under consideration. Finding out that the cycle length distributions do not preserve time-homogeneity, and that the Markovian property is not held on cycles. Results were corroborated applying conventional random walk tests: difference sign, individual and joint correlations. It is worth mentioning that the method of analysis introduced here involves measuring procedures rather than hypothesis testing, as detecting deviations from randomness is important for investors as it might help to improve the possibilities of obtaining profits.

Finally, we conclude that the random walk hypothesis does not hold in the IPC time series among the three periods.

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