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Macroeconomic Reverse Stress Testing: An Early-Warning System for Spanish Banking Regulators.[1]

Analysis Based on the 2008 Global Financial Crisis

Prueba de resistencia inversa Macroeconómica: una prueba de alerta temprana para los reguladores bancarios españoles.

Análisis basado en la crisis financiera global de 2008

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Abstract

This paper presents a methodology that helps regulators to identify early-warning alerts regarding the stability of the financial system. It is a macroeconomic Reverse Stress Testing analysis which examines the interrelationships between different factors in the financial system during an economic crisis period. Archimedean copulas (Gumbel copula) were applied in the modelling of these interactions, showing the interdependence of specific factors.

The methodology is applied using four factors: Bank loans to the insurance sector, Spanish exports, the Energy Price Index in Spain, and the growth rate of the Stock Price Index. First, each factor was projected for three years into the future. After that, each factor was calculated to identify the probability distribution that best fitted its projected data. Copula parameters were computed, and each alert level parameter for

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the financial system was established. Finally, an exhaustive analysis of the results was conducted.

JEL Classification: C22, C51, C53, G28, G32

***Keywords**: Reverse Stress Testing, Financial Stability, Time Series, Copulas.*

Resumen

Este documento presenta una metodología que ayuda a los reguladores a identificar alertas tempranas sobre la estabilidad del sistema financiero. Es un análisis de Pruebas de Resistencia Inversa macroeconómica que examina las interrelaciones entre diferentes factores en el sistema financiero durante un período de crisis económica. Se aplicaron cópulas arquimedianas (cópula de Gumbel) en el modelado de estas interacciones, mostrando la interdependencia de factores específicos.

La metodología se aplica utilizando cuatro factores: los préstamos bancarios al sector de seguros, las exportaciones españolas, el Índice de Precios de la Energía en España y la tasa de crecimiento del Índice de Precios de las Acciones. Primero, se proyectó el comportamiento futuro de cada factor por tres años. Luego, se analizó cada factor para identificar la distribución de probabilidad que mejor se ajustaba a los datos proyectados. Se calculó el parámetro de la cópula y se estableció el nivel de alerta de cada parámetro para el sistema financiero. Finalmente, se realizó un análisis exhaustivo de los resultados.

Clasificación JEL: C22, C51, C53, G28, G32

Palabras clave: prueba de resistencia inversa, estabilidad financiera, series de tiempo, copulas.

Introduction

The financial crisis that started in the U.S. in 2007 had an impact on the rest of the world which was felt from 2008 onwards due to the economic interlinkages between countries, which are intrinsic to the global economy. Well in advance of this crisis, Minsky (1975) posited the idea that the appearance of financial stability could lead to an untenable speculative euphoria. Therefore, achieving financial stability is crucial for the competent authorities, especially during an economic crisis. Nowadays such authorities are working on some macroeconomic methodologies in an effort to maintain financial stability and control speculation. One of these is the appropriately called *stress testing*, a useful method, designed by regulators and supervisors, to determine and analyse the level of the resilience of banks to adverse financial situations. These situations are hypothetical scenarios based on historical data. Breuer, Jandacka, Rheinberger, and Summer (2009, p. 206) state "*The quality of a stress test crucially depends on the definition of stress sce-*

narios"; and Dees, Henry and Martin (2017) argue that these scenarios must include sufficiently severe, yet plausible, financial crisis situations. Although this tool is widely used, it is also important to anticipate problems in the financial system that could potentially lead to financial instability. This supposes a complication in the definition of the scenarios to be considered in the stress tests where the resistance to financial stability is measured. One possible methodology for doing so is the so-called *reverse stress testing*. These types of methodologies are widely implemented at the individual level by the financial institutions themselves as part of a bottom-up approach (International Actuarial Association, 2013), but not at the global level of the entire financial system.

The purpose of this paper is to provide a reliable tool with which to anticipate alert levels in the Spanish financial system and to take prompt action on the risk factors identified as being the most vulnerable. The specific objective is to develop an early-warning system for the Spanish financial stability, based on the 2008 global financial crisis. Thus, a *reverse stress testing* methodology (RST) is applied as a possible way of establishing alert thresholds for financial stability in Spain. Dridi, El Ghourabi and Limam (2015) and Grundke and Pliszka (2013) analyse some limitations and weaknesses of this kind of models. (i) The bias in the selection of variables must be taken into account, so the experience and knowledge of the analyst is crucial for efficient management.; (ii) as Grundke and Pliszka (2013) postulate, this methodology is mathematically and conceptually demanding, given that for "n" risk factors, it is necessary to work with "n" dimensional scenarios, and for each individual scenario its probability of occurrence must be calculated; (iii) as Dridi, Ghouradi and Limam (2015) indicate the definition of the possible scenarios to analyze has the danger of ignoring some harmful, but plausible scenarios, and this can create an "illusion of security". It is important to take into account that the model proposed in this document is presented as a complementary analysis for financial stability, not as an isolated methodology. For the correct management of financial stability, several measures must be considered, including a macroeconomic reverse tension test, as proposed in this document. The following steps were used to develop the model: (i) the calculation of the dependence between factors through applying copula theory, (ii) the projection of each factor by means of analysing historical information, applying time series theory and then the selection of the marginal distribution that best fits the projected data for each factor; (iii) the estimation of the reverse scenario as early-warning indicators.

This methodology is validated by analysing if it could have predicted the last financial crisis; in order to accomplish this, the information of the quinquennial 2002-2006 was analysed and the factors projected for the following three years (2007-2009). The results show that in the projected values for that crisis, three of the four parameters included in the methodology reached an alert level. Moreover, one additional study has been included regarding the forthcoming situation of the Spanish financial system, in which data from years 2011-2016 is used to calculate alert levels for the period 2017-2019.

This paper has the following structure: it begins with the presentation of the methodological specifications, followed by an analysis of its validity. Finally, after computations are carried out, final results and conclusions are provided.

1. Methodological Specifications of Reverse Stress Testing

This paper proposes a methodology to helps regulatory authorities to establish early warnings alerts regarding the stability of the financial system, analysing different factors that impact on the financial system credit risk (i.e. on its solvency). For a given level of dependence between each factor, the methodology applied calculates the tail value of the solvency distribution.

1.1. Literature review

Different reverse stress testing methodologies are presented in the literature. The Value at Risk (VaR) methodology is a widely used technique for these estimations, but in general, it is based on the multivariate normality hypothesis (Yamai and Yoshiva, 2005). In reverse stress testing analysis, heavier tails are assumed, so it is useful to model distributions with fat tails, which is not the case of the hypothesis usually assumed in this type of analysis.

Dridi, El Ghourabi and Limam (2015) propose the application of a Reverse Stress Testing methodology based on the combination of risk factors, to find the worst case from which Tunisian banks become insolvent, focusing on credit risk. They used a methodology proposed by Wang, Peng and Yang (2013), who present a method to find the worst VaR, from a special kind of copula in which all the marginal distributions are identical and have a monotonous density (or monotonous distribution tail). This assumption generates loss of sensitivity for each behavioural factor included in the analysis.

From a VaR perspective, Kopeliovich, Novosyolov, Satchkov, and Schachter (2013) propose a methodology for reverse stress testing, applying the model of main components with Gram-Schmidt orthogonalization for the determination of scenarios that lead to the previously mentioned loss of sensitivity. They base their calculations on the VaR decomposition methodology to calculate five plausible scenarios. McNeil and Smith (2012) propose a methodology, which they call “Most Likely Ruin Event”, to solve the reverse stress testing problem. Grundke and Pliszka (2015) propose a quantitative method to apply the reverse stress testing methodology in banks exposed to credit and interest rate risk and demonstrate how the model can be calibrated. This is a useful model to calculate the reverse stress testing scenario for each bank, including macroeconomic variables as the systemic risk factors.

To avoid the problem of using historical data (such as time sensitivity), Cintas del Río (2007) proposes the use of copulas. The copula approach affords the opportunity to capture relationships between a multidimensional function distribution and its marginal factors. This method enables non-linear relations to be captured, focusing on extreme events (Casparri, García Fronti and Bianco, 2010).

As has already been mentioned, the published methodologies are diverse. It is necessary to adapt the method to the context in which the model is applied and the type of information available, taking the objective into account. The methodology applied in this paper takes into consideration the idea presented by Grundke and Pliszka (2015) at an individual, bank by bank level and takes into account the conclusions reached by Cintas del Río (2007) regarding copula theory. The objective is to achieve a methodology able to detect early warning levels from a list of factors for the Spanish financial system that indicate the risk of an impending new financial crisis due to the credit risk situation. Moreover, this paper uses aggregate information, taking into account different components of the system.

1.2. Specification of the methodology

The methodology proposed is based on the hypothesis that the most likely reverse stress scenario is associated with a high dependence between factors. This hypothesis is based on a Hull publication; he postulated that in times of crisis the interrelation between factors (dependence) tends to increase under extreme market conditions (Hull, 2006). To address this issue, this paper proposes a framework based on *Copula theory*. This theory links multivariate distribution functions to their marginal one-dimensional distribution functions (Nelsen, 2006), considering the dependence between the functions, and not only their linear correlation. With the factors selected, the first stage is to calculate the dependency between each factor through a copula parameter; the second stage is to forecast the behaviour of each factor and to select the best fit to the probabilistic distribution; the last stage is to obtain the reverse scenario searched through the calculation of the multivariate distribution function.

First stage: copula parameter

The parameter to be estimated in this step is usually called the dependency parameter of the copula, which measures the dependence between the marginals to be incorporated into the calculation. So, the objective is to calculate the dependency of each factor included in the analysis through the copula parameter; to this end it is necessary to consider the Sklar Theorem (Rüschendorf, 2013).

$$F(A_1, A_2, \dots, A_n) = C\big((F_1(A_1), F_2(A_2), \dots\dots, F_n(A_n)\big) \qquad (1)$$

Where F is a n-dimensional distribution function, C is a n-dimensional copula, F_x are the marginal distribution function of each factor, A_x and each factor, being x= 1, 2, …, n. If marginal distribution functions are continuous, then it is possible to define:

$$C(u_1, \dots, u_n) = P(F_1(A_1) \leq u_1, \dots, F_n(A_n) \leq u_n)$$

$$= P\big(X_1 \leq F_1^{-1}(u_1), \dots, X_n \leq F_n^{-1}(u_n)\big) \qquad (2)$$

$$= F_X\big(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)\big)$$

In the case of the methodology offered in this paper, the Gumbel copula is used because it is an Archimedean copula (to capture the dependencies in the tails of the functions) and extreme values copula (to model multidimensional structures with strange events).[2] Based on this, the formula to be solved is the following:

$$C_\theta(u) = exp\left(-\left(\sum_{i=1}^{n}(-\log u_i)^\theta\right)^{\frac{1}{\theta}}\right) \qquad \theta \in [1,\infty) \qquad (3)$$

Being θ the dependence parameter that should be calculated in this first step. A greater parameter should be interpreted as indicating a higher dependence between factors, and it should be parameterized by a real $\theta \geq 1$.

Taking into consideration both the aforementioned Hull postulation and that the objective is to anticipate a crisis after a stable period of time, this factor is stressed, including a higher value of its estimation instead of the mean. To do so, it was decided to apply the superior limit of the confidence interval at 99.9% in order to be aligned with the best practices in the estimation of risk protections in finance.

Second stage: projections and marginal probabilistic distribution selection

The next stage[3] is to project the available data for each factor included in the analysis, using historical information and estimating the projections. Autoregressive (AR), integrated (MA) and/or mobile average (ARIMA) series are used since the intention is to design a model capable of predicting a crisis alert while being in a stable macroeconomic and financial situation[4] (Enders, 1995). The analysis of seasonality and trend of each factor is carried out and these analyses are solved as required. The best adjustment is decided through the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), in combination with these two graphs: Autocorrelation (ACF) and Partial Autocorrelation (PACF).

After obtaining the projected data, it is necessary to decide the best fit between that information and a probability distribution. To this end, the descriptive statistics and the results of two hypothesis tests for each factor are taken into consideration: Shapiro-Wilk test (Shapiro and Wilk, 1965) and Kolmogorov-Smirnov (K-S) test (Chakravarty, Laha and Roy, 1967) testing

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2 *An additional contrast with other copulas distributions is presented in Annex 1.*

3 *First and second stages are independent, so it is possible to change their order.*

4 *If shocks are incorporated through the modelling of the time series (through GARCH or ARCH models, for example), then an additional assumption would be incorporated that could condition the possible identification of crisis alerts in stability situations.*

normal, chi-square, exponential, gamma, uniform, Weibull and Pareto distributions (Sarabia Alegría, Gómez Déniz, and Vázquez Polo, 2007). The final selection of the best fit is confirmed on graphical observation (temporal evolution, boxplot, histogram, and point graphs).

Third stage: obtaining the reverse stress scenario

Finally, the results of the previous steps are included in the multivariate distribution functions (including the copula parameter and the marginal distributions of each factor) to obtain the reverse scenario. This is the warning scenario that presents the highest probability of occurrence considering all previous assumptions.

2. *Results*

The next section presents the results applying the methodology explained. First, the validation of the tool is presented analysing the ability of the methodology to detect alerts for the previous financial crisis, using information until 2006. After that, the same stages are applied to analyse the possibility of a new financial crisis arising in the period of 2017-2020.

2.1 *Validation: estimation for the 2007-2009 triennium*

Variables in the period 2002-2006 are checked to detect if the methodology proposed had been able to anticipate the subsequent crisis (the following 3 years, 2007-2009).

The process of selecting variables to be incorporated in the final model involves different methodological proposals. Skoglund and Chen (2009) proposes a nonparametric method for extracting relative information from risk factors. The measure is based on Kullback information theory and indicates that it can be used to determine the relative importance of risk factors in defining gains and losses, which is useful for selecting the factors to be incorporated into the reverse stress testing model.

Licari and Suarez-Lledó (2012) present two methods to reduce the number of variables and factors: factor analysis (FA) and main component analysis (PCA). They stipulate that, once the number of factors has been reduced, RST calculations can be carried out with linear or non-linear models (such as logarithmic or logistical).

The factors selected in this process are based on the analysis by Cristófoli and García Fronti (2020). They proposed the application of Andon methodology to select factors that met the SMART requirements (Specific, Measurable, Archivable, Relevant and Time-bond). The techniques proposed select five factors reflecting the sensitivity of the Spanish financial system against credit risk, from a top-down perspective.

Taking into consideration that only positive associations are allowed between the variables for Archimedean copulas with a dimension of greater than 2 (Jaramillo-Elorza and Lozano, 2014), some further analysis and adjustments have been performed over the original list of factors, including the application of the rotation technique proposed by Patton (2012). Finally, four factors were selected to carry out the analysis (see Table 1).

Table 1. *Variables selected to incorporate into the final scenario*

Variable	Description
A1	Loans to the sector of insurance companies and pension funds of the Euro area.
A2	Series foreign trade in Spain (DXE). Exports provisional data deflected by the unit value index amount in Spain.
A3	CPI energy in Spain.
A4	Stock price index growth (IBEX 35).

Source: Author's calculation, based on the analysis carried out by Cristófoli and Fronti (2020) and data extracted from European Central Bank statistics (variable A1), Bank of Spain statistics (variables A2 and A3), Datastream (variable A4).

Note: This table provides the four variables included in the next analysis.

The final list of variables has four components, one showing the level of loans conceded by the total system to the insurance companies and pension funds (reflecting the level of investments that these types of companies are willing to undertake according to the economic perspective existing in this sector), and three macro variables, one reflecting the capacity of Spain to improve its balance of payments, the second presenting the level of industrial activity through energy prices, and the third showing the movement of consumer prices, a factor that has exerted significant influence during the latest crisis.

2.1.1. Copula parameter

The first stage in the analysis is to calculate the dependent parameter of the copula. To carry this out, the explanations given by Kojadinovi and Yan (2010) are considered. They set out the steps to be followed in the modelling of a copula with continuous margins in the software R, through the copula package. They indicate that this estimation of the parameter can be carried out through three methods: "mpl" (maximum pseudo-likelihood), "itau" (inversion of Kendall's tau), and "irho" (inversion of Spearman's rho). Pseudo-observations have been used[5] as in the case of the "mpl" method the use of pseudo-observations is a requirement. This transformation ensures that the observations all fall within the interval (0,1), without implying modifications to the behaviour of the variables.

The best method is chosen using the Cramer-von-Mises test (Anderson, 1962). The best estimation methodology for the correlation is determined among maximum pseudo-likelihood, Kendall's tau and Spearman's rho. The result, presented in Table 2, shows a very small p-value (less than 1%) in all three cases, so any of the methods would be valid.

Table 2. Three methods for the estimation of the Gumbel copula parameter

Method	Statistic	Parameter	p-value
itau	0,0653	1,1230	0,0008
irho	0,0679	1,1127	0,0003
mpl	0,0671	1,1153	0,0056

Source: Author's calculation, based on projected data (2007-2009) for the four factors included in the analysis.

Note: This table provides the results of the estimation of the Gumbel copula parameter for the projected data (2007-2009) of the selected variables using three different estimation methods: itau, irho and mpl.

The method selected is "mpl", aligned with the recommendation by Kojadinovic and Yan (2010), who indicate that this estimation method should be used for estimation of copulas with dimensions greater than 3 in the Gumbel family.

[5] In the following steps the pseudo-observations of the projected data for the years 2007, 2008 and 2009 are used.

The parameter estimated would indicate the interrelation between the variables in a regular scenario. But the objective pursued is to determine the interrelation of the variables in an adverse scenario. At those times, as discussed earlier, the interaction between the variables is greater. The verification of this assumption is presented in Table 3, where the calculation of the copula parameter is performed with the information of the real data from 2009 to 2012 (financial and economic crisis period). As it is possible to see, the factor with the real data is greater than that calculated previously. For this reason, it was decided to compute the 99.9% percentile and assign that value to the final estimation. In this way, the possible contagion is taken into account between the different sectors incorporated into the analysis (through the incorporated factors), which would be greater in extreme market conditions (Hull, 2006).

Table 3. Gumbel copula parameter estimation

	Factors projected (2007-2009)	**Real information (2009-2012)**	
	Alpha	Percentil 99.9	Alpha
Estimate	1,115	1,2015	1,279
Std. Error	0,028		0,082
Maximized loglike lihood	14,02		7,198

Source: Author's calculation, based on projected data (2007-2009) and real data (2009-2012) for the four factors included in the analysis.

Note: This table provides the results of the estimation of the Gumbel copula parameter for the projected data (2007-2009) and real data (2009-2012), standard error of this estimation and maximized loglikelihood, and additional include the percentile 99.9 of the projected date.

2.1.2. Projections and marginal probability distributions selection

The methodology applied at this stage has three steps: first, the analysis of the most relevant characteristics of each factor; second, the behaviour of each variable was projected through the time series tool; third and finally, a probability distribution that best fits the projected variables was selected for each factor.

The descriptive statistics are presented in Table 4 and graphical analyses have been carried out to begin the analysis of the factors.

The second step starts with the graphical view of each factor, presented in Figure 1, which is complemented with the Augmented Dickey-Fuller Test to determine if the seasonal adjustment of each series is necessary.

Table 4. Descriptive statistics on the four factors

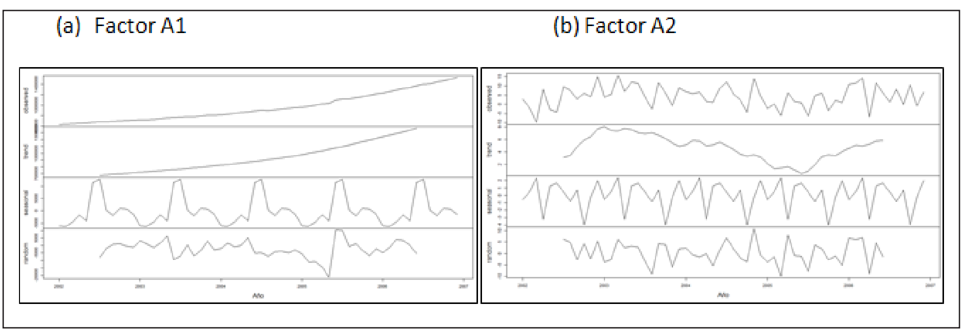
Variable	Min	1stQu.	Median	Mean	3rd Qu.	Max.
A1	630.224,00	1.160.394,00	1,441.702,00	1.417.266,00	1.816.158,00	1.896.9110,00
A2	22,49	0,18	4,97	4,21	8,73	24,66
A3	15,900	1,900	2,600	3,210	10,750	21,400
A4	0,00778	0,00108	0,00043	0,00013	0,00160	0,00755

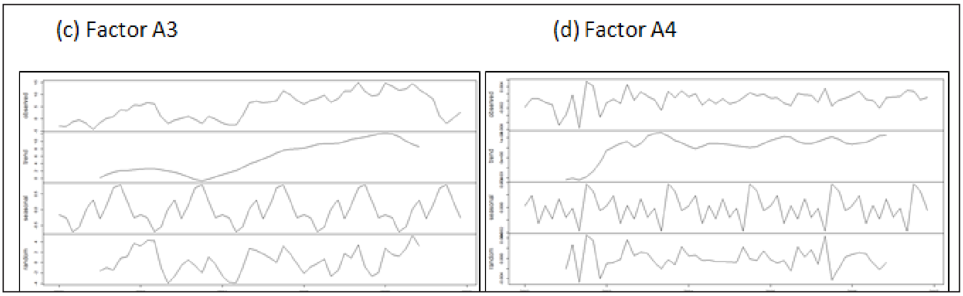
Source: Author's calculation, based on data extracted from European Central Bank statistics (variable A1), Bank of Spain statistics (variables A2 and A3), Datastream (variable A4).

Note: This table provides the minimum, 1st quarter (P25), median (P50), mean, 3rd quarter (P75) and maximum for each factor.

After all necessary adjustments (including the trend) were made, between 8 and 10 time series were analysed for each factor, and the best adjustment was selected based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) tests, combined with graphical analysis (autocorrelation and partial autocorrelation graphs).

Figure 1. Breakdown of the time series for each factor, monthly data from January 2002 to December 2006





Source: Prepared by authors

Nota: The first line of each quadrant shows the observed data, the second, the trend of the data, the third, the season of the data and the fourth and last one, the random data.

Finally, the probability distribution function that best fits the projected factors was selected based on Shapiro-Wilk normality test and Kolmogorov-Smirnov test, presented in Table 5, contrasted with the QQ-plot graphs, presented in Figure 2.

Table 5. Shapiro-Wilk and Kolmogorov-Smirnov tests results

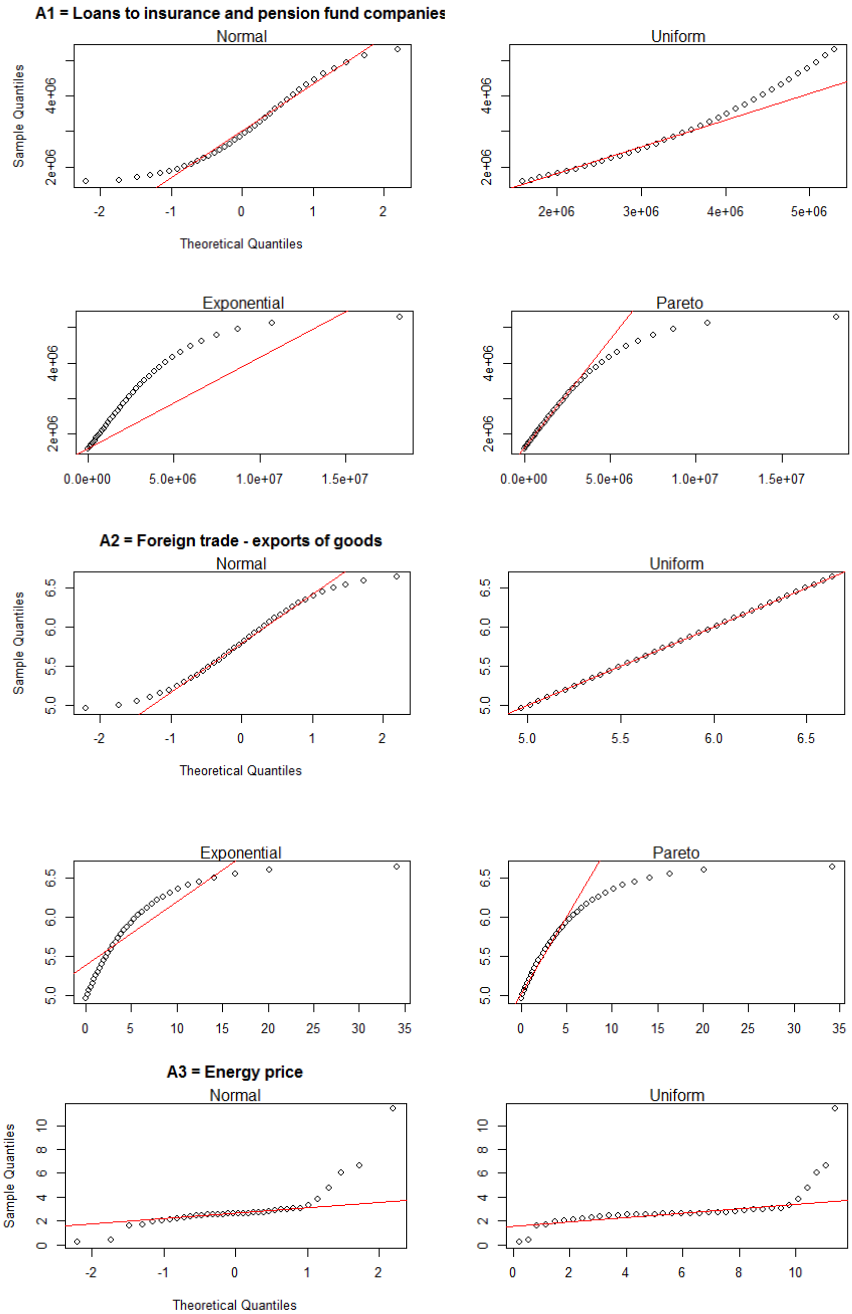
Factors	Normal				Chi-square		Exponential	
	S-W normality test		K-S test		K-S test		K-S test	
	W	p-value	D	p-value	D	p-value	D	p-value
A1	**0.939**	**0.046**	**0.099**	**0.460**	0.556	0.000	0.180	0.087
A2	0.957	0.169	0.071	0.987	0.476	0.000	0.575	0.000
A3	0.664	0.000	0.315	0.001	0.211	0.034	**0.191**	**0.064**
A4	0.953	0.129	0.071	0.662	0.993	0.000	0.517	0.000

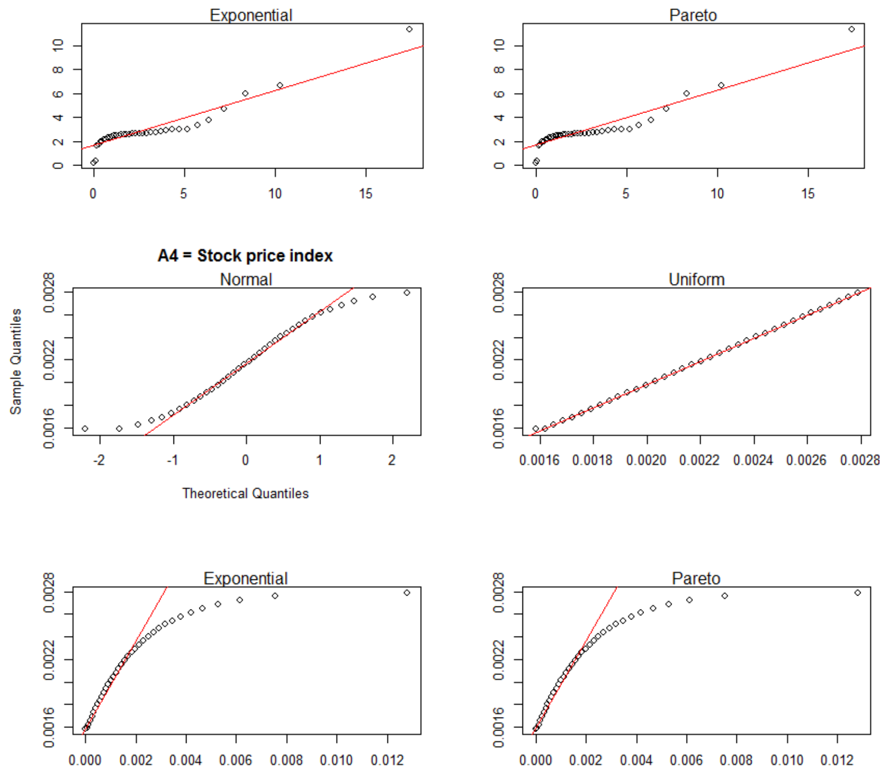
Factors	Gamma		Uniform		Weibull		Pareto	
	K-S test		K-S test		K-S test		K-S test	
	D	p-value	D	p-value	D	p-value	D	p-value
A1	0.077	0.621	0.161	0.139	0.089	0.536	0.179	0.088
A2	0.074	0.981	**0.028**	**1.000**	0.083	0.947	0.575	0.000
A3	0.238	0.014	0.582	0.000	0.257	0.007	0.191	0.064
A4	0.079	0.605	**0.028**	**0.928**	0.071	0.663	0.517	0.000

Source: Author's calculation, based on data projected for each factor.

Note: This table provides results and p-value of Shapiro-Wilk normality test and Kolmogorov-Smirnov test for distributions Normal, Chi-square, Exponential, Gama, Uniform, Weibull and Pareto for each factor. The final decision is indicated in bold for each factor.

Figure 2: QQ-plot for each factor on four of the analysed distributions, including Normal, Uniform, Exponential and Pareto





Source: Prepared by authors

Finally, the specifications of the marginal distributions to be included in the calculation of the reverse scenario for each factor are detailed in Table 6.

Table 6. Marginal distributions

Variable	A1	A2	A3	A4
Probability distribution	*Normal*	*Uniform*	*Exponential*	*Uniforrm*
Parameters	mean 3085944 sd 1093443	min 4.961651 max 6.640721	rate 0.3383459	min 0.0015819722 max 0.0027900910

Source: Author's calculation based on previous calculations.

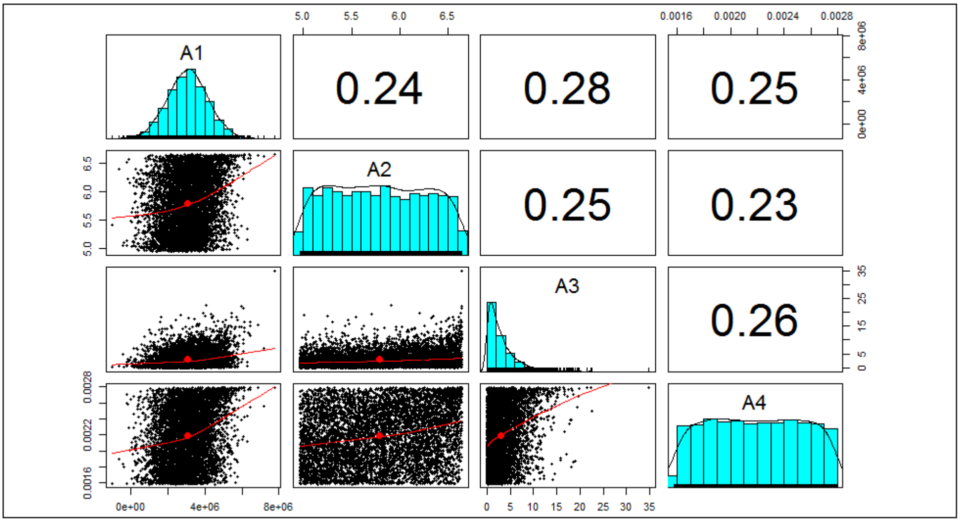
Note: This table provides the probability distribution function chosen to project the values of the next years and the parameters of each probability function.

2.1.3 Obtaining the reverse stress scenario

The last stage of the validation process is to obtain the reverse stress scenario. As the calculation of a crisis scenario for the factors is required (a severe scenario but at the same time sufficiently probable, after which the financial system could enter into a crisis), the average was selected for each variable with which to identify alerts. In this way, the selected scenario would begin to generate concerns among the economic and regulatory agents and could lead to a crisis due to the interaction between the economic sectors.

Figure 3 shows the simulated values calculated through the copula, considering the selected marginal and the parameters calculated in previous steps.

Figure 3: Pair plot of the random variables for a Gumbel copula



Source: Prepared by authors.

Table 7 shows the average of the estimated values (mean), that is, the scenario that has been presented most frequently in the simulation of the marginal distribution projections and considering the dependency between factors through the calculation of the copula's parameter. This value, the mean, is considered the early-warning limit for each factor.

Table 7. Descriptive Statistics on parameter projections (marginal) after copula calculation

	Mean	Min	Max	P(25)	P(50)	P(75)	P(95)
A1	1,378,109	NA	8,910,986	2,394,975	3,088,439	3,824,544	4,884,912
A2	5.80	4.96	6.64	5.38	5.80	6.22	6.56
A3	2.954	0.000	49.592	0.849	2.045	4.098	8.875
A4	0.00218	0.00158	0.00279	0.00189	0.00219	0.00249	0.00273

Source: Author's calculation, based on previous estimations.

Note: This table provides the mean, 5th, 50th and 95th percentiles (p5, p50, p95) for the four variables included in the estimation.

The regulator would be alerted if the selected variables approached values above those of the mean scenario. This analysis with 2006 information is presented in Table 8.

Table 8. Backtesting analysis result

Variable	2016 data	Early warning limit	Analysis Result
A1	1,378,109	3,085,000	
A2	6.08	5.8	ALERT
A3	8.230	2.954	ALERT
A4	0.00112	0.00218	ALERT

Source: Author's calculation, based on information from European Central Bank statistics (variable A1), Bank of Spain statistics (variables A2 and A3), Datastream (variable A4) and previous calculations.

Note: This table provides information from 2006 for each variable and the previously calculated value as early-warning limits.

Table 8 presents the identification of the crisis alerts for the period 2007-2009, comparing the 2006 values of the factors with the means included in Tables 7, it can be seen that three of the four factors exceed[6] the threshold established as a limit value.

6 The last factor is below the threshold because this variable had to be rotated to meet the requirements of Gumbel copulas, among which is that all factors must have a positive relationship.

By interpreting these results, it is possible to see the validity of the methodology presented. Thus, the next step is to apply the presented methodology with the information available up to 2016, in order to see the risk of a new crisis occurring for the period 2017-2019.

2.2. Projection: estimation for the 2017-2019 triennium

This section analyses the possibility of a new crisis, considering the same variables incorporated in the previous analysis, with information from the period 2011-2016; the values for the next three years are projected -that is, 2017 to 2019-, finally the early warning limits are obtained to be compared with the real information of 2016. The final results are presented in Table 9.

Table 9. Results of the estimated scenario for 2017-2019 and comparison with 2016 information

Variable	2016 data	Early warning limit	Analysis Result
A1	1,291,220	1,547,000	
A2	3.75	4.76	
A3	-8.420	0.400	
A4	0.00005	0.00218	ALERT

Source: Author's calculation.

Note: This table provides the information from 2016 for each variable and the value calculated as early-warning limits.

There is still an alert identified but this corresponds to the growth in the price index, which is highly affected and correlated to the growth in the housing price index. This sector has a very slow recovery process because it depends on the industrial structure to be able to prosper. Changes in credit for the whole of the financial sector are transmitted to the macroeconomy with a delay which is quantified as being between two and four quarters (Gerba and Mencia, 2017).

The results presented up to the previous paragraphs are based on the preselected set of non-related factors. However, the alert identification methodology presented allows factors to be incorporated that are considered arbitrarily relevant, even though they are closely related to each other.

An analysis of this type would be extremely valid if it were developed in order to compare the results obtained with the scenarios created in a stress testing analysis in which the same variables were incorporated. Additionally, the interaction of these values with those proposed in a stress testing scenario would help to analyse the validity of the shocks included in the scenario designed for the stress testing analysis.

2.3. The application of the model of arbitrarily selected factors

The results presented up to now are based on a preselected set of factors. However, the alert identification methodology proposed allows incorporating factors that are considered relevant arbitrarily, even though they are highly related to each other. To demonstrate this, the results of the estimation for the 2017-2019 triennium is presented below. This variables are generally considered in the stress testing scenarios in Spain: A5 (year-on-year variation rate of the Gross Domestic Product), A6 (year-on-year variation of social benefits due to unemployment) and A7 (stock price index growth). In this case, the result of the analysis is shown in Table 10.

Table 10. Results of the estimated scenario for 2017-2019 and comparison with 2016 information for three new variables

Variable	2016 data	Early warning limit	Analysis Result
A5	0.3050	0.3765	
A6	- 10.84	0.0000193	
A7	0.00005	0.00217	

Source: Author's calculation.

Note: This table provides the information from 2016 for each variable and the value calculated as early-warning limits.

As can be seen, the stock price index growth does not give an alert value, as it did in the result presented in section 3.2. The result estimated as alert level in the most probable scenario is slightly different (0.00217 vs 0.00218). This shows a possible contagion between them. This is a different interpretation that one in Table 9, remarking the importance of carrying out a previous analysis of the selection of variables. A poor selection of factors can generate a lack of alert identification, as in this case.

An analysis of this type is useful to compare the results obtained with the scenarios created in a stress testing analysis in which the same variables are incorporated. The interaction of these values with those proposed in a stress testing scenario shall help to analyse its validity.

Conclusion

The proposed methodology has proved to be useful for identifying crisis alert levels for the leading financial factors that affected Spain from 2008 onwards, and the calculations carried out for the future situation enabled the detection of an alert level for the triennium 2017-2019.

As for the backtest performed, the procedure yielded alerts for three of the four studied parameters. The crucial parameter was the stock price index, presumably because the beginning of the crisis in Spain was characterised by a price bubble, particularly in housing and developer loans. The other two parameters were energy prices, which portray industrial activity swings, and the export of goods, which reflects problems in foreign trade operations.

Concerning the forecasting test, conducted with the same variables and procedures, the methodology yielded alert levels for the triennium 2017-2019. The fundamental parameter, in this case, was the price index growth, which is profoundly affected by and correlated with the growth in the housing price index in Spain. This sector tends to experience very slow recovery processes due to its dependence on the industrial structure. The results obtained show that such a process has been taking place in Spain since the last great crisis. The methodology presented in this work is expected to be useful in the future, contributing to prevent future financial crises, at best; or at least, diminishing the devastating impact that these have on society.

The results obtained proved the robustness of the methodology applied here. However, three main weaknesses call for new lines of future research. First, the bias in the selection of the variables must be considered. New research on the Spanish financial sector could focus on different variables to the ones used in this work. Second, the data was projected with ARIMA Time Series; thus, both the methodology and the data are preconditioned by the fact that shocks are not taken into consideration. Hence, this methodology enables the detection of alert levels during periods of financial stability without taking into account any interaction with past crises, that is, assuming that historical data does not necessarily mirror or grant insight into possible future crises. Finally, it is possible to predict alert levels for potential crises through this methodology, but only from a credit-risk perspective and provided that other risk factors such as political risk, remain constant. Nevertheless, such technical limitation can be overcome by utilising other types of time series models such as GARCH or ARCH.

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Anexo 1

The goodness-of-fit tests is carried out in order to strengthen the decision of the Gumbel copula's selection for the calculation of the reverse scenario. To this so, the method Kendall´s tau is used to perform the comparison. The results are presented in the Table A.1.

Table A.1. Four options for the estimation of the copula parameter

Method	Statistic	Parameter	p-value
Gumbel	0,0653	1,1230	0,0008
Frank	0,0890	0,9397	0,0000
Clayton	0,1321	0,2460	0,0000
tCopula	0,0701	0,0298*	0,0004

Source: Author's calculation, based on projected data (2007-2009) for the four factors included in the analysis.

Note: This table provides the results of the estimation of the copula parameter for the projected data (2007-2009) of the selected variables using four different copulas: Gumbel, Frank, Clayton and tCopula.

*Full results for tCopula are: parameter1 = 0.029837, parameter2 = 0.325490, parameter 3 = 0.069562, parameter4 = 0.343040, parameter 5 = 0.081633, parameter 6 = 0.093052

As can be seen, even though the highest p-value is obtained with Gumbel copula, it is too small. For that reason Gumbel copula is selected, but another method is applied to estimate the correct parameter. The details are presented in section 3.1.1. Additionally, it is selected because it has the properties of the Arquimedian copulas and the properties of the extreme value copulas (Naifar, 2011).