

Estimation of Alpha in Defined Benefit Pension Funds with a t-Student O-GARCH Matrix. A test in Pensiones Civiles del Estado de Michoacán*

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Estimación de alfa en fondos con beneficios definidos mediante una matriz t-Student O-GARCH. Una evaluación de las pensiones civiles del Estado de Michoacán.

Resumen

En este artículo se evalúa la utilidad de un proceso de administración activa de portafolios empleando una matriz de covarianzas GARCH ortogonal (O-GARCH) con función de verosimilitud t-Student, al aplicarlo en la reserva técnica de fondos de pensiones de beneficio definido de la Dirección de Pensiones Civiles del Estado de Michoacán. Esto tanto para lograr el objetivo de 7.5% anual de rendimiento establecido en su estudio actuarial como para definir *alpha* respecto al *benchmark* establecido en su política de inversión. Para demostrar esto, se corrieron tres simulaciones de eventos discretos en donde se ejecutó, en una de ellas, un proceso de administración pasiva de portafolios con una disciplina de rebalanceo de tipo *posición objetivo* y en las otras dos una activa de tipo *rebalanceo por rangos*. Con los resultados observados se resalta la preferencia de utilizar, en este fondo de pensiones, la administración activa de portafolios con la matriz O-GARCH.

Clasificación JEL: C12, G11, G12, G17

Palabras Clave: Selección de portafolios, Valuación de activos, Pronósticos y simulaciones financieras, Pruebas de hipótesis.

Abstract

This paper presents an assesment of an active portfolio management process with a t-Student orthogonal GARCH (O-GARCH) covariance matrix, in order to achieve a 7.5% actuarial target return and to formulate alpha in defined benefit pension funds for Dirección de Pensiones Civiles del Estado de Michoacán. To test this, three discrete event simulations were performed using, in the first one, a passive portfolio management process with a target position rebalancing discipline and, in the other two, an active portfolio management with range portfolio rebalancing where an equally weighted covariance and a t-Student O-GARCH covariance matrix are used. The results suggest that the O-GARCH matrix is suitable for active portfolio management in this sort of pension funds.

JEL classification code: C12, G11, G12, G17

Key words: *Portfolio Selection, Asset Pricing, Financial forecasting and Simulation, Hypothesis testing.*

1. Introduction

There are several pension fund schemes in Mexico. Among them, the defined benefit and the defined contribution are the most common. In the former scheme, a percentage of the salary earned over the last year as an employee at the time of retirement. In the latter, the employee retires only with the amount of money saved during the accumulation period,¹ leaving the pension fund with no other responsibility than to deposit into and, in some cases, to manage the account. A detailed review of all the pension fund schemes used in Mexico and their main financial reform proposals are outside the scope of this paper. For the interested reader, a very straightforward review is given by the Instituto Mexicano de Ejecutivos de Finanzas (2006).

In each state of Mexico, there are several pension funds that manage the retirement savings for their public servants. Among these, a defined benefit pension fund, which is owned by the public servants of Michoacán and known as “Dirección de Pensiones Civiles del Estado de Michoacán (DPCEM)”, is the focus of this paper.

The DPCEM covers all the public servants in the State of Michoacán (about 15,000). Among the most observable legal liabilities in the pension plan is that all the beneficiaries retire with the 100% of its salary paid during the last year in service. In order to fund this, the Government and the employees save an amount equal to the 4% of the employees' monthly wages, considering an actuarial yearly wage increase of 6.5%. By assuming a life expectancy of 81 years for Mexico and due to the fact that an employee in this pension fund can retire after 30 years of service (an average 45-50 years of age at retirement) and a 4% theoretical inflation rate, it is necessary for DPCEM either to achieve a nominal 7.5% yearly return in the investment policy of its Technical Reserve (TR)² or to change the monthly contributions from 4% to 11% of the employees' salaries.

¹ Before retirement.

² A trust created to support the pension plan when the outflows are higher than the inflows (about 2032 with the current scheme).

This paper focuses on the first proposal and its aim is to show the usefulness of an active portfolio investment process that uses a t-Student log likelihood O-GARCH covariance matrix in the TR of DPCEM. The main focus is to test an active portfolio management process with the historical asset allocation in the six different markets shown in the investment policy statement (IPS) presented in Appendix one. A benchmark that incorporates this asset allocation and its limits is also given in this appendix along with the target investment positions (\mathbf{w}_{bmk}) related to it.

Why an active portfolio management process? With the advent of the Markowitz (1952) model, the portfolio management practice started a notable development with a “buy and hold” rationale that led to a portfolio management practice known nowadays as “*Passive portfolio management*”. Considering the Markowitz-Tobin-Sharpe-Lintner (MTSL) model (Markowitz, 1987, p. 5) and the assumption of an aggregate optimality due to homogeneous expectations among investors (Samuelson, 1965) —that leads to the concept of market equilibrium (Sharpe, 1963; Lintner, 1965)—, the stock market indexes are used not only as a statistical measure of the aggregate investor behavior but also as a proxy of the market portfolio, a key concept in the main asset pricing models.

Due to several economical, financial and behavioral circumstances, the aggregate optimality (as a proxy definition of *equilibrium* in financial markets) is not observable in the short term, suggesting the preference of *active portfolio management*. This situation has been improved with the development of time series analysis through the seminal works of Box, Jenkins and Reinsel (2008) and later with the proposals of Engle (1982) and Bollerslev (1986) for the short term volatility forecast. With these quantitative developments, the Portfolio Management Theory had a positive and practical advance which allowed to explore and exploit short term price differentials, leading to support the active management practice. Several research papers have been published in order to test active portfolio management against the passive approach in the mutual fund industry with cases such as Daniel *et al.* (1997) and Ennis (2005) who found, through the mutual fund comparison against a stock index after management fees, that the active management practice couldn't lead to a better performance than a passive one such as *index tracking* or *enhanced index tracking*.³

³ Index tracking means that the manager must replicate the behavior or (if possible) the conformation of a market benchmark or index. This practice could lead to

In some cases (such as *index tracking*), the passive management strategy can be executed by following a *Target Positions* (TP) portfolio rebalancing discipline, where the portfolio position is rebalanced to that of the benchmark's (\mathbf{W}_{bmk}). In contrast, the active management practice, which seeks to outperform a benchmark or a market index, could be executed with two rebalancing disciplines (among the most used) known as *Percentage Portfolio Rebalancing* and *Range Portfolio Rebalancing* (RPR). The former is a rebalancing method executed at periodically specified time intervals where the portfolio manager adjusts the investment positions to a range of $\pm x\%$ from a target optimal position \mathbf{W}_{bmk} ; the latter consists of discretionary investment proportions that must follow upper and lower asset or market type limits, stated in an IPS like the one presented in Appendix one.

From several strategies widely used as rebalancing disciplines in the portfolio management practice and from those previously mentioned, DPCEM decided to test an RPR discipline using the IPS shown in Appendix one as a legal and institutional mandate. This situation allows the fund manager to invest in a relatively discretionary manner in different types of assets allowed in the IPS.

In order to assess whether a passive Target Position (TP) or an active Range Portfolio Rebalancing (RPR) portfolio management strategy is more suitable to the fund, three discrete event simulations were performed. One was performed for the passive portfolio management case with a TP regime and two for the RPR active portfolio management that use two different covariance matrixes: 1) a constant or equally weighted covariance and 2) an O-GARCH with a t-Student log likelihood one.

Once established the aim of the present paper (to test the use of active portfolio management with O-GARCH matrixes in the Technical Reserve of DPCEM and similar ones) the results will be presented as follows: In a second section, a brief explanation of the Markowitz-Tobin-Sharpe-Lintner model is given along with a review of the O-GARCH covariance matrix model. Following this, the assumptions and general structure of the algorithm used in the three discrete event simulations are presented along with the results obtained. After this, the document ends with the concluding remarks.

some limitations such as the impact of financial costs (trade fees, market timing, tax impact or liquidity) therefore the enhanced index tracking discipline tries to achieve higher gross returns than the replicated benchmark in order to compensate the impact of financial costs.

2. The optimizer used in the active portfolio management process.

2.1 The Markowitz-Tobin-Sharpe-Lintner portfolio selection model.

The Markowitz-Tobin-Sharpe-Lintner or MTSL model is a theoretical proposal that incorporates a risk-free asset in the asset allocation, a drawback that the original Markowitz (1952) *standard* model couldn't solve. In the MTSL, the optimal portfolio selection is now conceived as a linear combination of a risk-free asset rf and a risky portfolio given by \mathbf{w}^* . This selection is performed in a two-step problem that starts with the solution of the next optimization problem given an $n \times 1$ investment proportion vector \mathbf{w} , an $n \times n$ covariance matrix \mathbf{C} , an $n \times 1$ expected return vector $\mathbf{r} = [E(\mathbf{r}_1) \ \dots \ E(\mathbf{r}_n)]'$, an $l \times n$ asset or market grouping matrix \mathbf{D} and an $l \times 1$ minimum or maximum limits vector \mathbf{d} established in the IPS:⁴

$$\mathbf{w}^* = \arg \max_{\mathbf{w}} [\mathbf{w}' \cdot \mathbf{r} - (rf \cdot \mathbf{1})'] \cdot [\mathbf{w} \cdot \mathbf{C} \cdot \mathbf{w}]^{-1/2} \quad (1)$$

Subject to:

$$\mathbf{w}^* \cdot \mathbf{1} = 1$$

$$\mathbf{w}^* \geq 0$$

$$\mathbf{D} \cdot \mathbf{w}^* = \mathbf{d}$$

The second step is given by the proportion in the total investment budget ω in \mathbf{w}^* and the proportion $(1 - \omega)$ in rf determined, following Levy and Markowitz (1979), with a quadratic utility function in the next expression:⁵

$$\omega = \arg \max_{\omega} [\omega \cdot (\mathbf{w}' \cdot \mathbf{r})] - \omega^2 \cdot 0.5 \cdot A [\mathbf{w}' \cdot \mathbf{C} \cdot \mathbf{w}] \quad (2)$$

⁴ Such as the one described in appendix one.

⁵ In the case of Pensiones Civiles del Estado de Michoacán, a value of $A = 4$, related to a "neutral risk aversion investor" is set.

Once ω is determined, the final optimal portfolio is a linear combination of rf and the *risky asset* \mathbf{w}^* :

$$\mathbf{w}^{**} = \omega \cdot \mathbf{w}^* + (1 - \omega) \cdot rf \quad (3)$$

The portfolio selection model in (3) is the so-called Markowitz-Tobin-Sharpe-Lintner model or MTSL (Markowitz, 1987, p. 5) and is one of the most widely used in the portfolio management industry (as is the case in this paper) because it incorporates a risk-free asset in the asset allocation step. Although the rationale of the MTSL is a very straightforward one, its main drawback is observed in the calculation of \mathbf{r} and \mathbf{C} due to the presence of estimation error; its computational inefficiency and also the presence of volatility and correlation clustering. As a potential solution for the computational efficiency problem, Sharpe (1963) proposed an alternative calculation of such parameters that led to the proposal of the CAPM model (Sharpe, 1966).⁶ This valuation model is the theoretical foundation of several heuristics and alternative portfolio selection models that have, as a central concept, the covariance of assets now proxied through the covariance with a common factor or set of factors. Although the CAPM model is a straightforward rationale for asset pricing, and setting aside the theoretical critiques made against it, the presence of volatility and correlation clustering and the potential presence of estimation error are some of its main drawbacks as is the general case in the models of the Modern Portfolio Theory. Therefore, a pure mean-variance framework was adopted that led to choose the MTSL as the optimizer for the asset allocation step in the portfolio management process.

Best and Grauer (1991) suggested that the optimal portfolio selection is sensitive to the sample magnitude observed in \mathbf{r} and \mathbf{C} , the aim of this paper is to assess the performance of an active portfolio management process using the t-Student Orthogonal GARCH (O-GARCH) matrix, in defined benefit pension funds such as DPCEM. Given this, it is necessary to review this multivariate volatility model and the need for a t-Student log likelihood function.

⁶ This in an almost parallel approach to Lintner's (1965) proposal.

2.2 The Orthogonal GARCH model (O-GARCH) for the calculation of the covariance matrix.

With the earliest proposals of Engle (1982) and Bollerslev (1986), the calculation of short term volatilities that incorporate the volatility clustering effect, allowed the financial practice to forecast risk levels more appropriately. The univariate $GARCH(p, q)$ model follows an expression shown by:

$$\sigma_t^2 = \kappa + \sum_{i=1}^p \beta_i \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \cdot \sigma_{t-i}^2 \quad (4)$$

As a starting point for this paper, the $GARCH(p, q)$ model departs from the assumption that the returns vector \mathbf{r}_i of the i^{th} asset is either⁷ $\mathbf{r}_i | I \sim N(\bar{\mathbf{r}}_i, \sigma_{\mathbf{r}_i}^2)$ or $\mathbf{r}_i | I \sim t(\bar{\mathbf{r}}_i, \sigma_{\mathbf{r}_i}^2, gl)$, leading to the more general assumption of multivariate elliptic probability functions in the set of returns time series $\mathbf{X} = [\mathbf{r}_1, \dots, \mathbf{r}_i]$. With this, the log likelihood problem can be solved through two functions. If $\mathbf{r}_i | I \sim N(\bar{\mathbf{r}}_i, \sigma_{\mathbf{r}_i}^2)$ the vector of parameters $\theta_i = [\kappa, [\beta_i], [\gamma_i]]$ leads to⁸ $\sigma_t^2 = \sigma_t^2(\theta_i)$ in (4) and the optimal set of parameters θ^* is shown with the solution of the next optimization problem:

$$\arg \max_{\sigma_t^2(\theta_i)} = \frac{1}{2} \sum_{t=1}^T \left[\log(\sigma_t^2) + \left(\frac{\varepsilon_{i,t}}{\sigma_t} \right)^2 \right] \quad (5)$$

Subject to:

$$\begin{aligned} \kappa, \beta_i, \gamma_i &\geq 0 \\ \sum_{i=1}^p \beta_i + \sum_{j=1}^q \gamma_j &\leq 1 \end{aligned}$$

When $\mathbf{r}_i | I \sim t(\bar{\mathbf{r}}_i, \sigma_{\mathbf{r}_i}^2, gl)$, the solution is shown by the next log likelihood function maximization⁹:

⁷ Given the information set I_{t-1} that makes r_t conditional.

⁸ $\varepsilon_i = r_i - (\bar{r}_i \cdot 1) = [\varepsilon_{i,t}]$

⁹ Please see Bollerslev (1986) and Lambert & Laurent (2001).

$$\arg \max_{\sigma_t^2(\theta_i)} = - \sum_{t=1}^T \left\{ \left[\log(\sigma_t^2) + \left(\frac{gl+1}{2} \right) \cdot \log \left(\frac{1}{gl-2} \cdot \left(\frac{\varepsilon_t}{\sigma_t} \right)^2 \right) \right] + \right. \\ \left. T \cdot \log \left[\frac{\Gamma \left(\frac{gl+1}{2} \right)}{\left((gl-2) \cdot \pi \right)^{1/2} \cdot \Gamma \left(\frac{gl}{2} \right)} \right] \right\} \quad (6)$$

Subject to:

$$\kappa, \beta_i, \gamma_i \geq 0 \\ \sum_{i=1}^p \beta_i + \sum_{j=1}^q \gamma_j \leq 1 \\ gl \geq 2$$

The expression in (6) will be used by the DPCEM based on the theoretical assumption that the t-Student matrix is more suitable to model sample probabilities and more appropriate to measure the characteristic fat tails of the financial data used.

For the multivariate case, one of the first proposals is the one made by Bollerslev (1990), which starts with the use of a constant correlation matrix **H** and a diagonal matrix **S** defined by univariate GARCH variances:

$$\mathbf{C}_t = \mathbf{S}_t \cdot \mathbf{H}_t \cdot \mathbf{S}_t \quad (7)$$

Despite its low computational efficiency, this model does not take into account the correlation clustering effect. A model that solves this situation is the BEKK GARCH of Engle and Kroner (1993), but, in some cases, this represents a hard computational task given the log likelihood maximization problem inherent to it. As a solution for this situation, a model known as *Orthogo-*

nal GARCH, *O-GARCH*(p, q) or simply *O-GARCH* is proposed in Alexander and Chibumba (1996), Alexander (2002), and a generalized version in Van der Weide (2002). This model departs from the spectral decomposition of an equally weighted¹⁰ covariance matrix \mathbf{C}_c that leads to the definition of a $n \times n$ matrix of eigenvectors \mathbf{E} and a $n \times n$ spectrum $\mathbf{\Lambda}$:

$$\mathbf{C}_c = \mathbf{E} \cdot \mathbf{\Lambda} \cdot \mathbf{E} \quad (8)$$

The computational efficiency of the *O-GARCH* model is based on the variance (eigenvalues) of the principal components ($\mathbf{P} = \mathbf{X} \cdot \mathbf{E} = [\mathbf{p}_1, \dots, \mathbf{p}_i]$) in the diagonal elements of $\mathbf{\Lambda}$. Once this matrix is defined, a selection of principal components, eigenvalues and eigenvectors, is made by sorting the eigenvectors and principal components from the highest (h) to the lowest eigenvalue. Following this, the next selection criteria is applied, given a total variance explanation level ν (percentage) previously fixed:

$$\lambda_i \in \mathbf{\Lambda}^*, \mathbf{e}_i \in \mathbf{E}^*, \mathbf{p}_i \in \mathbf{P}^* \Leftrightarrow \sum_{h=1}^i \frac{\lambda_h}{\text{trace}(\mathbf{\Lambda})} \leq \nu \quad (9)$$

With the definition of $\mathbf{\Lambda}^*$, the calculation of a univariate GARCH volatility is made in each main component in \mathbf{P}^* by using the log likelihood function given in (6). This will lead to the definition of a GARCH spectrum $\mathbf{\Omega}$ and to the next matrix composition of the expected *O-GARCH* covariance matrix:

$$\mathbf{C}_{\text{OGARCH}} \approx \mathbf{E}^* \cdot \mathbf{\Omega} \cdot \mathbf{E}^{*'} \quad (10)$$

Why use this specific multivariate GARCH model? There are two reasons: a) computational efficiency, b) its practical usefulness in financial risk modeling to calculate high dimension matrixes with low latency data.

¹⁰ An equally weighted covariance matrix is given by:

$$\mathbf{C} = \left[\left(\left[\mathbf{I} - \frac{1}{n} \cdot \mathbf{1} \cdot \mathbf{1}' \right] \cdot \mathbf{X} \right)' \cdot \left(\left[\mathbf{I} - \frac{1}{n} \cdot \mathbf{1} \cdot \mathbf{1}' \right] \cdot \mathbf{X} \right) \right] \cdot \frac{1}{n}, \mathbf{X} = [\mathbf{r}_1, \dots, \mathbf{r}_i]$$

The computational efficiency of the O-GARCH model can be compared against the Engle and Kroner (1993) BEKK-GARCH model and a previous and more general proposal of Bollerslev (1986) known as Vech-GARCH. In the latter case, the number of parameters to be estimated is given by the n number of factors with the following expression: $\left[n(n+1)(n(n+1)+1) \right] / 2$. In the BEKK-GARCH it is necessary to calculate $n(n+5)/2$ parameters and in the O-GARCH model, as noted in (9) and (10), it is necessary to calculate at most $\pi \cdot p \cdot q$ parameters where π is the number of main components selected with (9), p the maximum ARCH lag term allowed in the univariate GARCH of each main component model and q the maximum GARCH lag term.

Several papers can be mentioned related to the practical usefulness of the O-GARCH, Alexander (2002) that presents the calculation of large covariance matrixes with different kinds of assets like currencies, UK gilt bonds term structure, English equities, and the oil futures term structure, noting that the limitation of quality and amount of data¹¹ and the “dimensionality course”¹² can be avoided thanks to the main components analysis inherent to this multivariate GARCH model.

Another practical use is reviewed by Bredin and Hyde (2004) who test several covariance models, such as the equally weighted¹³, the exponentially weighted; the O-GARCH and the O-EWMA,¹⁴ in the Irish FX market previous to Ireland’s integration to EMU. Their results supported the O-GARCH model as the best one for capital reserve purposes and the O-EWMA for compliance.

Following Bredin and Hyde, Cifarelli and Paladino (2004, 2006) use the O-GARCH model to test the contagion of credit default events in the behavior of sovereign bond term structures in Latin America and Asia. They found, thanks to the O-GARCH model ability to capture the correlation and volatility clustering effect with the lack of data, that there is a weak integration in the Asian and Latin American sovereign bond markets in low volatility time periods, but this tends to disappear with the presence of contagion after specific market shocks such as the 2001 Argentinean default.

¹¹ A property observed in fixed income assets or futures term structures.

¹² I.e. that the number of assets could lead to a flatter log likelihood function.

¹³ Please refer to note 10.

¹⁴ Also proposed in Alexander (2002).

Also, Chen *et al* (2008) and Kearney and Muckley (2008) use this multivariate model to quantify either the Chinese stock market Value at Risk or the pegging effect (high and stable correlation and volatility) in the main Asian markets and currencies.

Therefore, because of its computational efficiency, O-GARCH model's ability to measure the correlation clustering effect even if the historical data is not latent,¹⁵ and its several practical applications such as the usefulness to quantify a measure of financial markets integration, to identify possible currencies that could potentially be pegged, and to quantify large covariance matrixes of heterogeneous financial assets by avoiding the "dimensionality course", it is the most appropriate for measuring the volatility of a portfolio invested in the six financial markets stated in the IPS given in Appendix one. In fact, its usefulness is still researched nowadays in extensions such as the one made by Sharifi *et. al.* (2012) where M-estimators are tested to calculate more robust GARCH parameters with less stringent moment conditions.¹⁶

Now that the calculation of the MTSL model and the O-GARCH covariance matrix have been reviewed as parts of the optimizer used in the portfolio management process, the assumptions of the three discrete event simulations performed are presented, noting that a proof of the presence of volatility and correlation clustering in the six benchmarks of the investment universe is shown in Appendix three.

3. The discrete event simulations performed.

3.1 Statistical parameters, theoretical assumptions and practical implications in each simulation.

Given a time frame from January 2nd, 2002, to December 31st, 2010, 470 weekly interval simulations are performed for each simulated portfolio, using each of the benchmarks presented in Appendix one as financial assets.¹⁷

These financial assets or benchmarks were valued at Mexican pesos (MXN) at a December 29th, 2000 base 100 value and incorporated currency impact. The length of each time series (\mathbf{r}_i) is $T = 52$ weeks and it is assumed

¹⁵ I.e. the financial asset price does not change due to a lack of liquidity.

¹⁶ At this point the out of sample and robust estimation scenario is set aside.

¹⁷ Assuming that these values represent the behavior of zero tracking error Exchange Traded Funds (ETF's) invested in each benchmark.

that these represent the behavior of zero tracking error Exchange Traded Funds (ETF's) that replicate them.

A quantitative analysis algorithm that performed the entire portfolio selection process (analysis, rebalancing and mark to market valuation) was programmed in MATLAB and, among the most relevant ones, the following assumptions and parameters were considered:

- 1) The theoretical¹⁸ starting value of the four simulated portfolios is 10,000,000.00 MXN, using the inflows and outflows presented in Appendix 2.
- 2) The financial data sources are Bloomberg™, Reuters™ and Infosel^{MR}.
- 3) In order to incorporate the impact of financial costs, a 0.25% fee is assumed in each trade either in the ETF's or in the FX market (noting that an institutional investor such as DPCEM can get access to a lower transaction cost). This fee will be used in order to measure a higher impact in the final turnover results.
- 4) The risk-free asset rf used is the weekly secondary market curve rate of 28-day-maturity Mexican treasury certificates (CETES). This rate was published on 2012 by Banco de México.
- 5) Only an MXN bank account and two investment contracts (one in US dollars and another in MXN pesos) will be used. When a foreign asset position (USD valued) is sold, the amount is turned into Mexican pesos by selling USD using the current FX rate. When the opposite happens, the US dollar amount is funded from the Mexican bank account.
- 6) The expected values in the return vector \mathbf{r} are shown in the following expression:

$$\forall E(\mathbf{r}_i) \in \mathbf{r}, E(\mathbf{r}_i) = \bar{\mathbf{r}}_i = \sum_{i=1}^T r_i \cdot n^{-1} \quad (11)$$

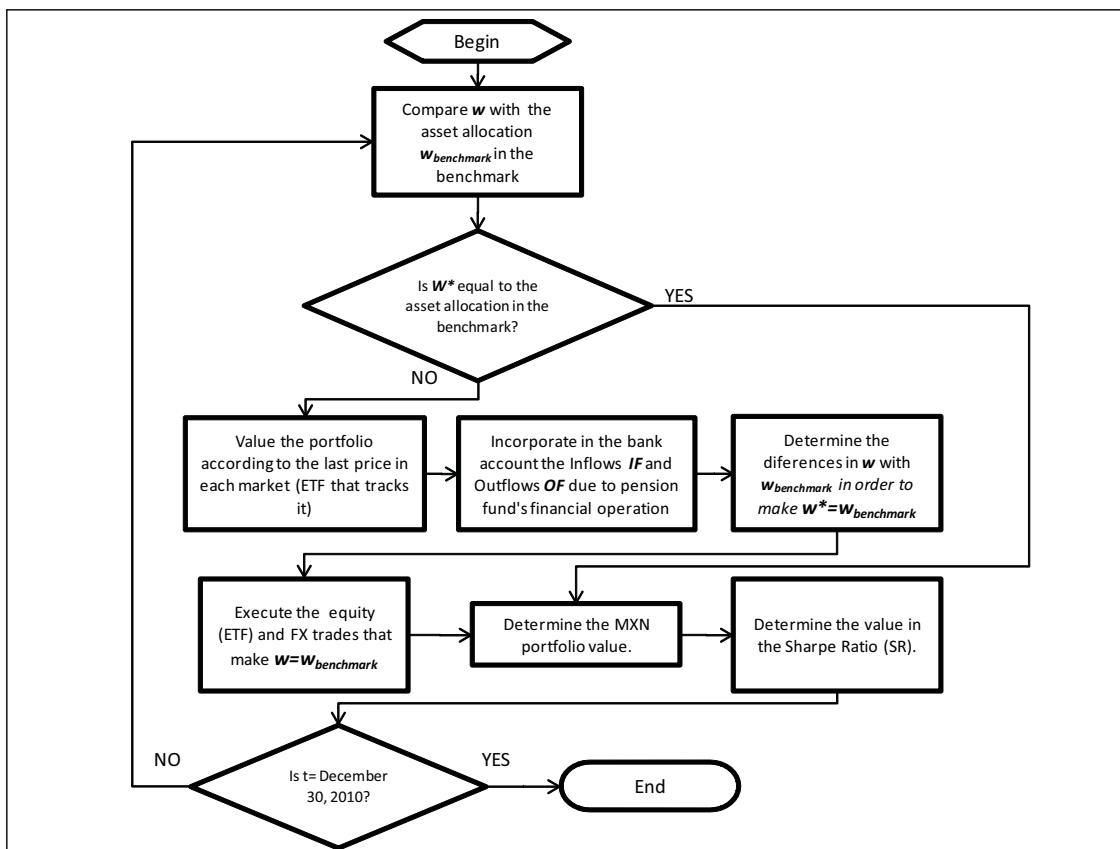
In order to calculate the O-GARCH matrix with (10) using (6) as the log likelihood function, the algorithm selected the best $GARCH(p, q)$ model for each

¹⁸ The original value of the pension fund was modified to MXN\$ 10 million due to confidentiality issues.

main component by using different ARCH lag terms truncated at the value of five and different GARCH lag terms truncated at the value of two. The goodness of fit of the best GARCH model in each principal component is measured by the Bayesian information criterion of Schwarz (1978).

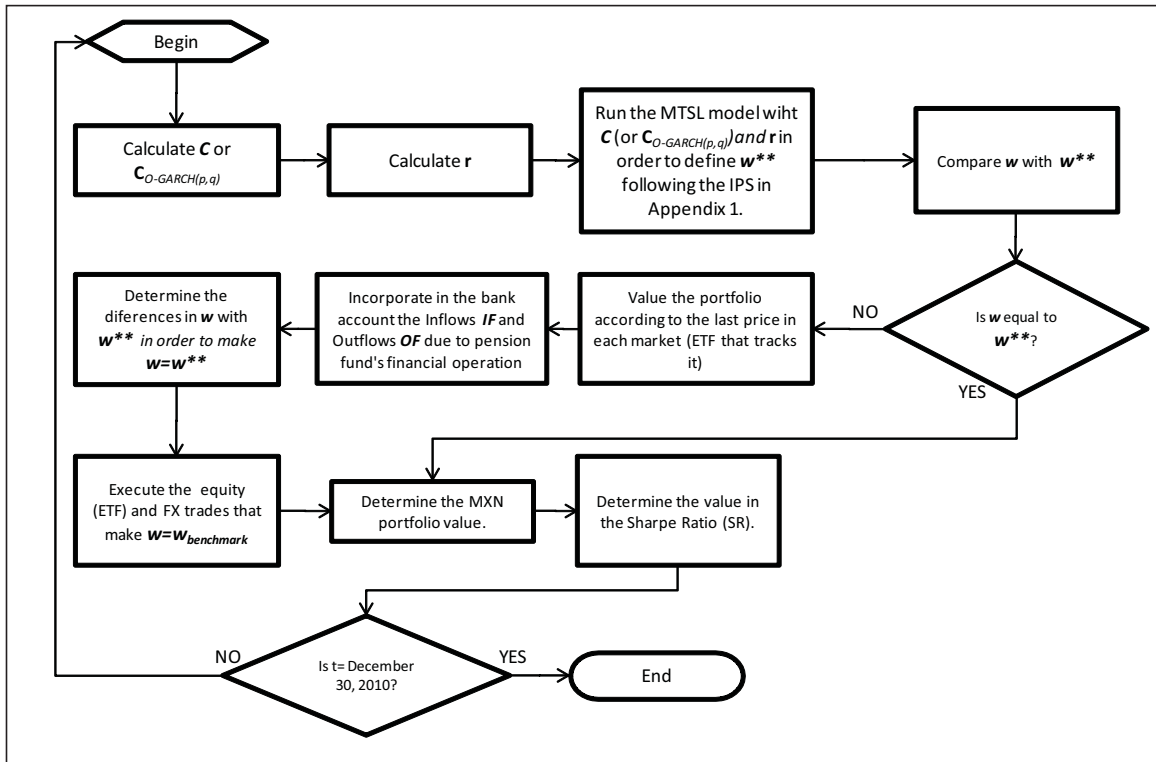
For the passive management (TP) portfolio simulation, the main assumption is that all the investment balance is allocated in the risky asset given by the benchmark asset allocation ($\mathbf{w}^* = \mathbf{w}_{\text{bmk}}$) shown in Appendix one. In order to rebalance from the actual investment proportion, a 0.25% financial cost is also incorporated and the algorithm shown in Figure one is performed.¹⁹ In order to execute the two discrete event simulations in the active portfolio management simulations, the algorithm of figure two was also used. Finally, with the results obtained, the three historical simulated portfolios were valued in a January 2nd, 2002, base 100.

Figure 1. Flowchart of the discrete event simulation performed in the passive portfolio management ("Target position").



¹⁹ As noted, this is an index tracking passive portfolio management practice.

Figure 2. Flowchart of the discrete event simulation performed in the active portfolio management (“Range portfolio rebalancing”) using the three different covariance matrixes.



3.2. Results observed in simulations.

The historical value of the simulated portfolios and their accumulated turnover is presented in chart one and summarized in table one. It is shown that the three simulated portfolios and the benchmark had a better performance than a theoretical financial asset that paid the 7.5% target return (light area). As shown, the simulated portfolios using the O-GARCH covariance matrix lead to a superior turnover than the benchmark, the passively managed and the constant parameter covariance matrix portfolios.

In order to confirm this result and to follow the portfolio management performance evaluation practices, a quality chart of the difference between the observed weekly return of each simulated portfolio and the benchmarks is presented in Chart two. As noted, the O-GARCH portfolio showed the highest positive alpha against the benchmark, suggesting a better performance if an O-GARCH matrix is used in the active portfolio management.

Chart 1. Performance comparison of the four simulated portfolios.

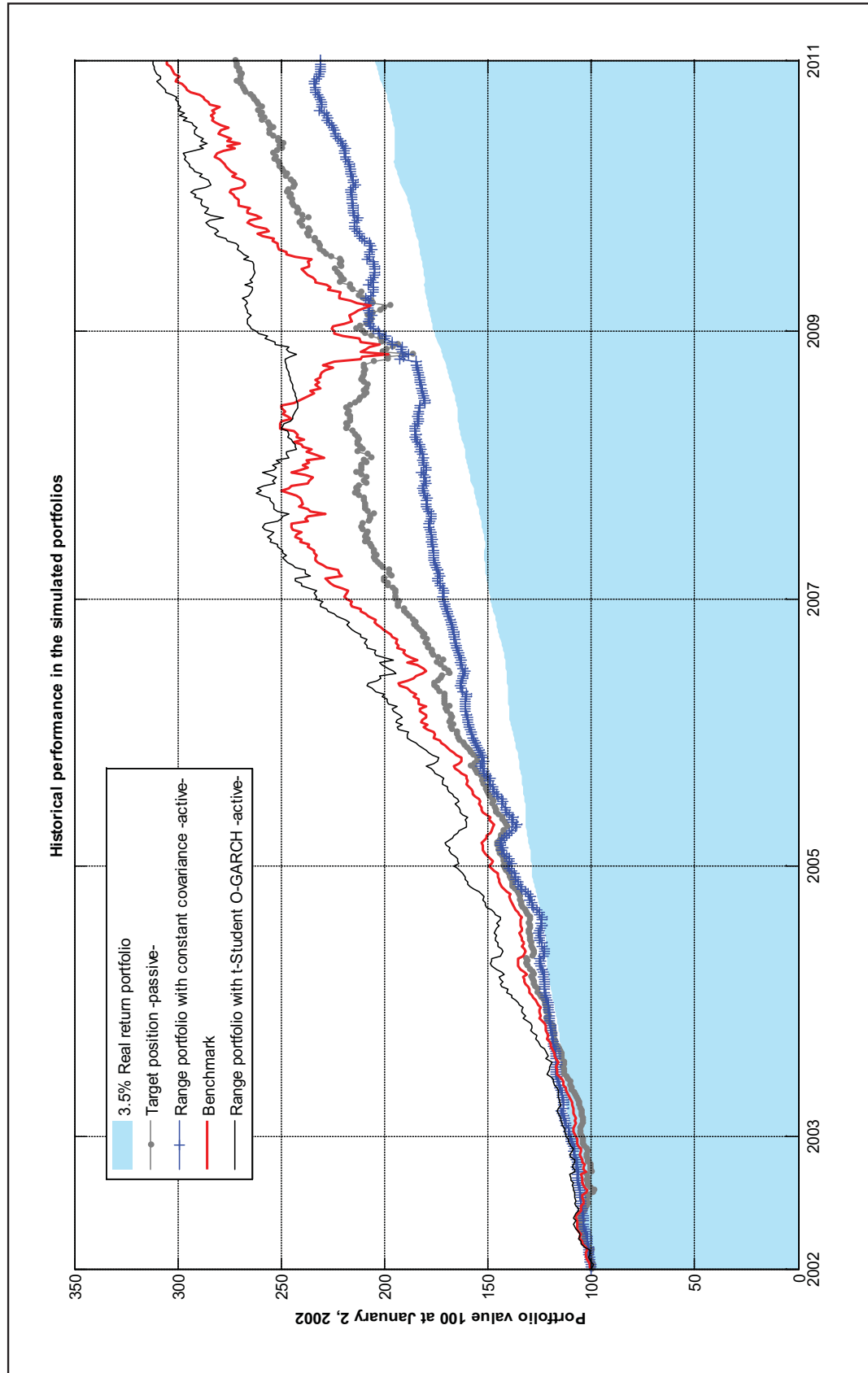


Table 1. Accumulated turnover in the four simulated portfolios.

Portfolio or benchmark	Accumulated turnover	Yearly effective return
7.5% actuarial target return	105.05%	11.67%
Benchmark	205.34%	22.82%
<i>Passive management: Target Position</i>	172.12%	19.12%
<i>Active management: constant covariance matrix.</i>	131.13%	14.57%
<i>Active management: Gaussian OGARCH covariance matrix</i>	210.14%	23.35%
<i>Active management: t-Student OGARCH</i>	212.22%	23.58%

A more detailed examination of the results obtained during the three simulations is presented in Chart three where the historical allocation between the risk-free asset rf and the risky portfolio \mathbf{w}^* can be observed. The reader should note that the portfolios simulated with an O-GARCH covariance matrix (specifically the t-Student one) were more sensitive in the risk-free asset investment proportion during the dates where the financial crisis was acute (e.g. the Lehman Brothers Chapter eleven filing in the September-October 2008 period). This is due to the fact that the volatility and correlation clustering effect²⁰ was measured more accurately in this period, leading to a higher concentration in the risk-free asset for longer time periods in comparison with the other two portfolios.

Another perspective of these results is shown with a complete historical asset allocation in Chart four. In the case of the O-GARCH covariance matrix portfolio, the optimizer manages more accurately the investment in riskier markets such as the Mexican equity (IPC index) or the foreign equities proxied with the MSCI Global Gross equity. This historical behavior is summarized in the box plots of chart five that shows the different investment levels in each asset type for each portfolio.

It should be noted that the passive portfolio and the active one that use the equally weighted covariance matrix were highly concentrated in the Mexican government bond and international treasury bond markets (especially the former), suggesting that even though the IPS presented in Appendix one

²⁰ In order to confirm that the level of volatility clustering was high in certain time periods like the aforementioned one, please refer to the historical ARCH test results shown in Appendix two.

Chart 2. Quality chart of the four simulated portfolios against the benchmark proposed in table one.

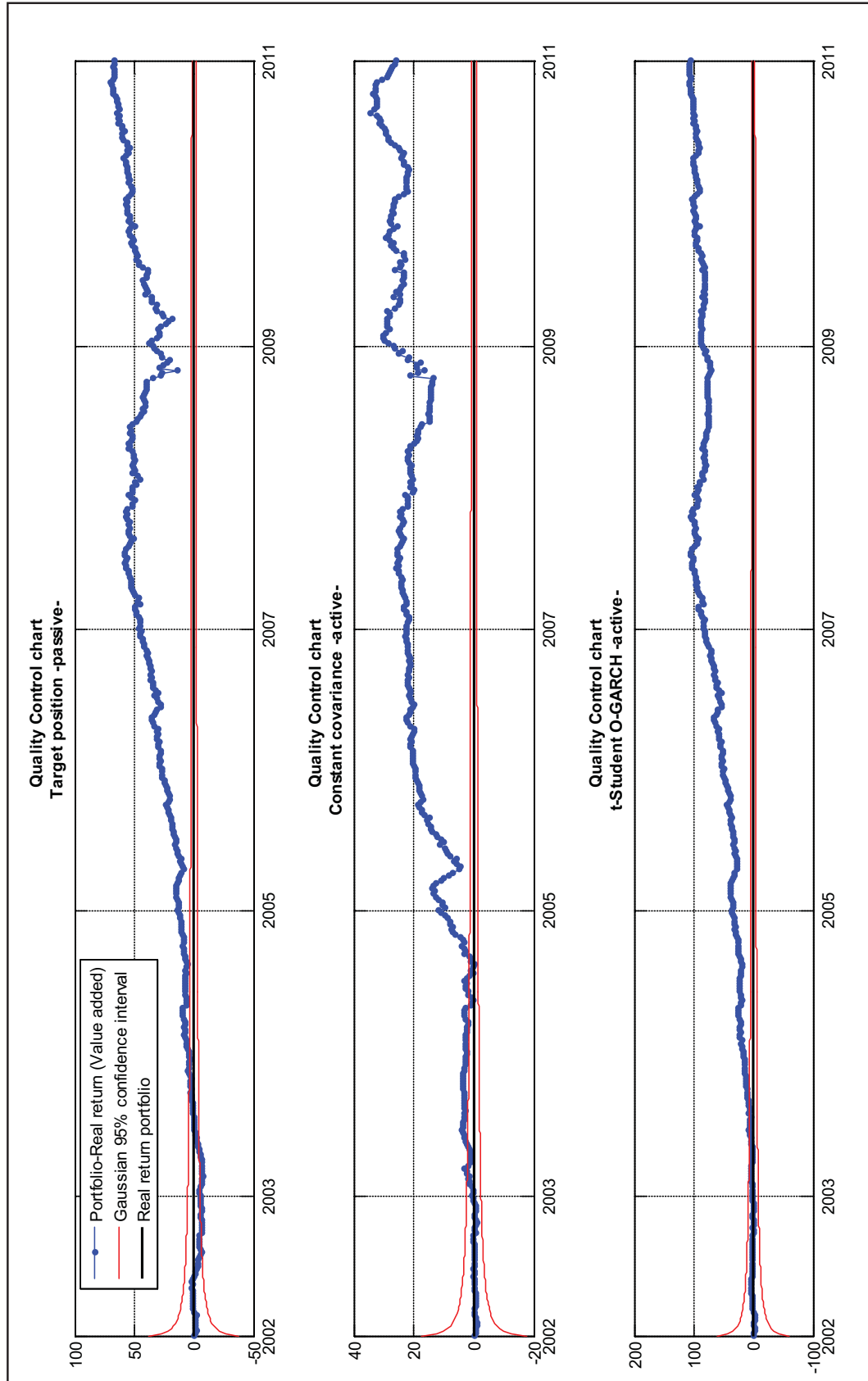


Chart 3. Historical investment proportions (risk-free asset v.s. entirely risky diversified portfolio) in each simulated portfolio.

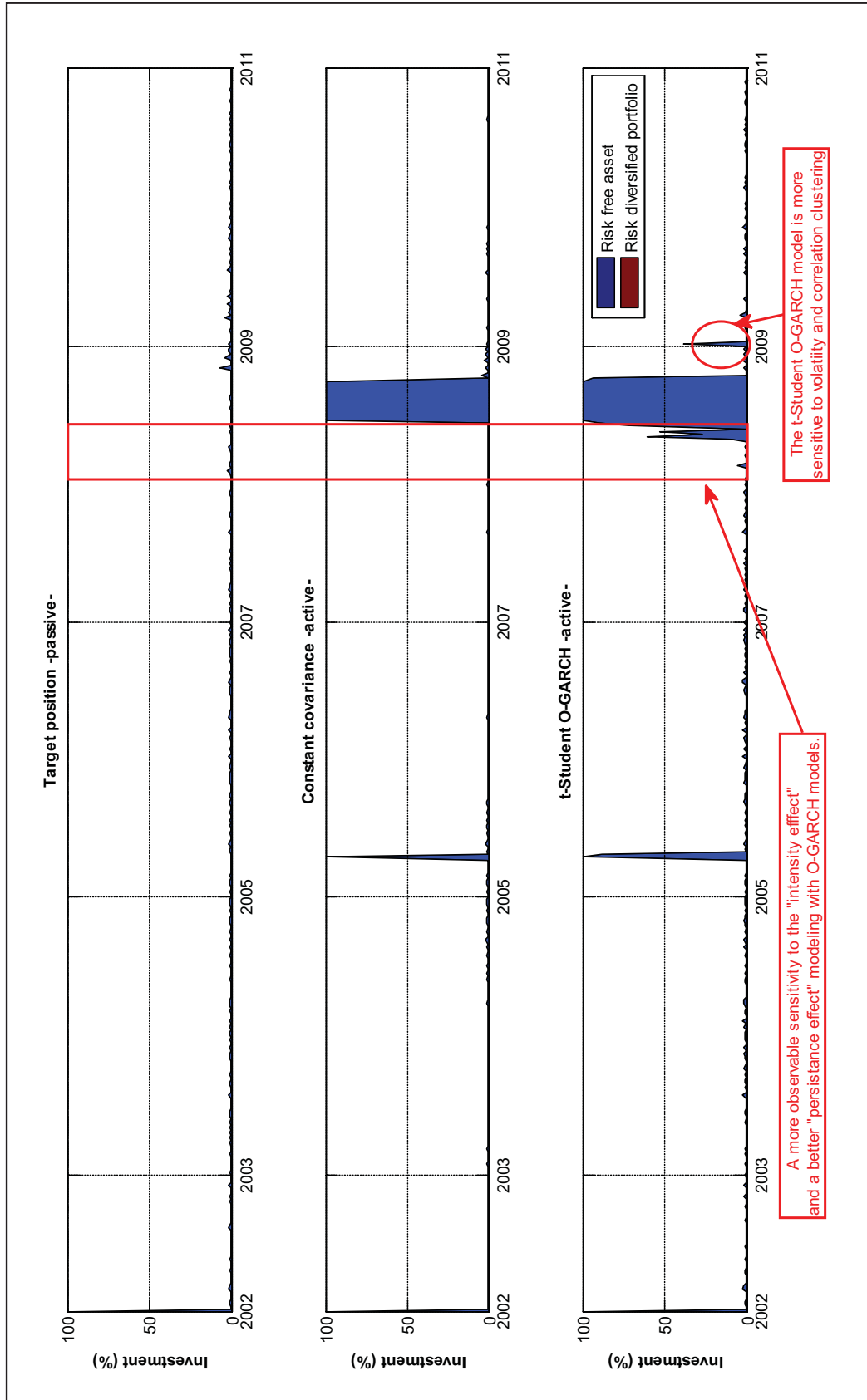


Chart 4. Historical investment proportions in the entirely risky portfolio.

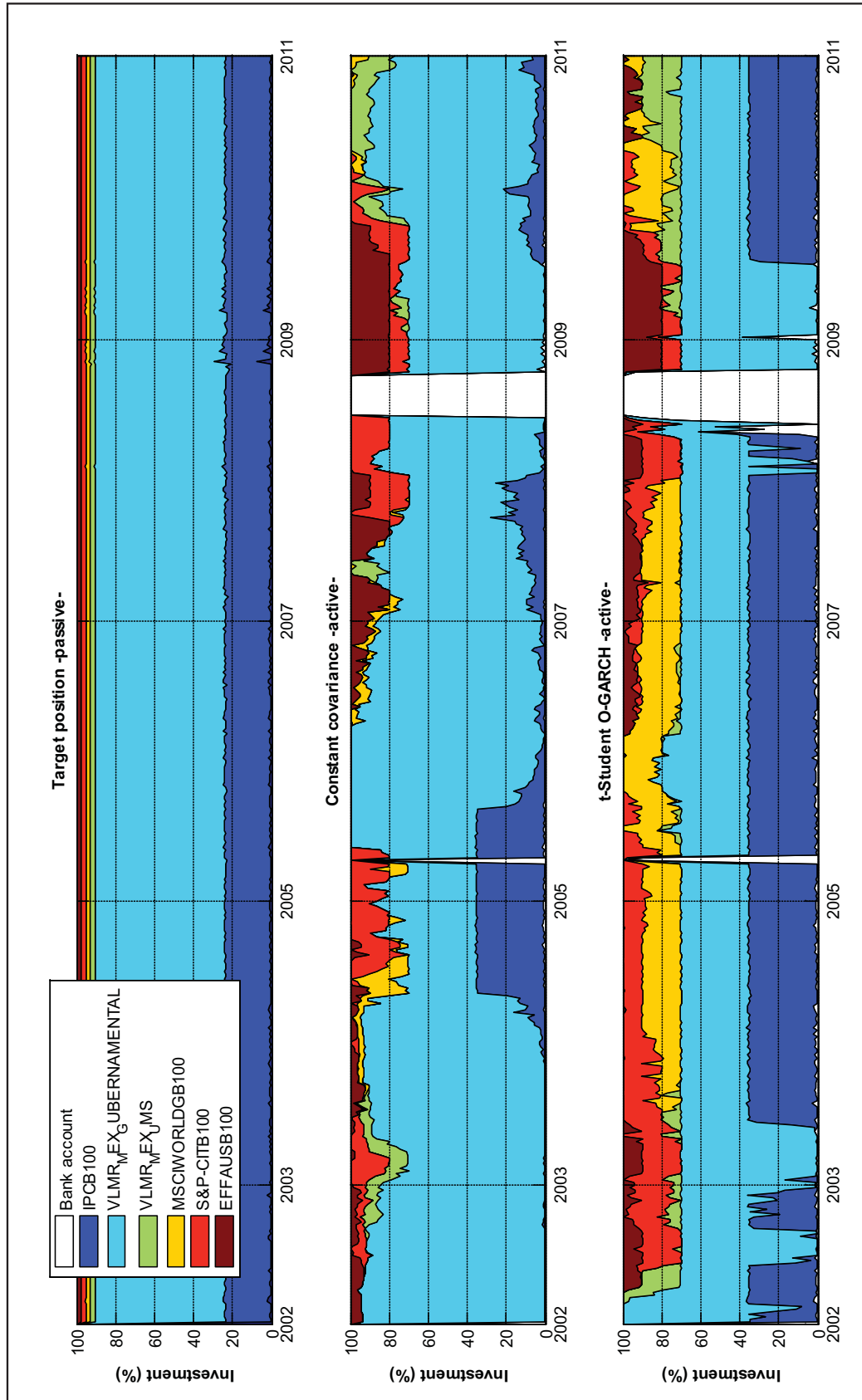
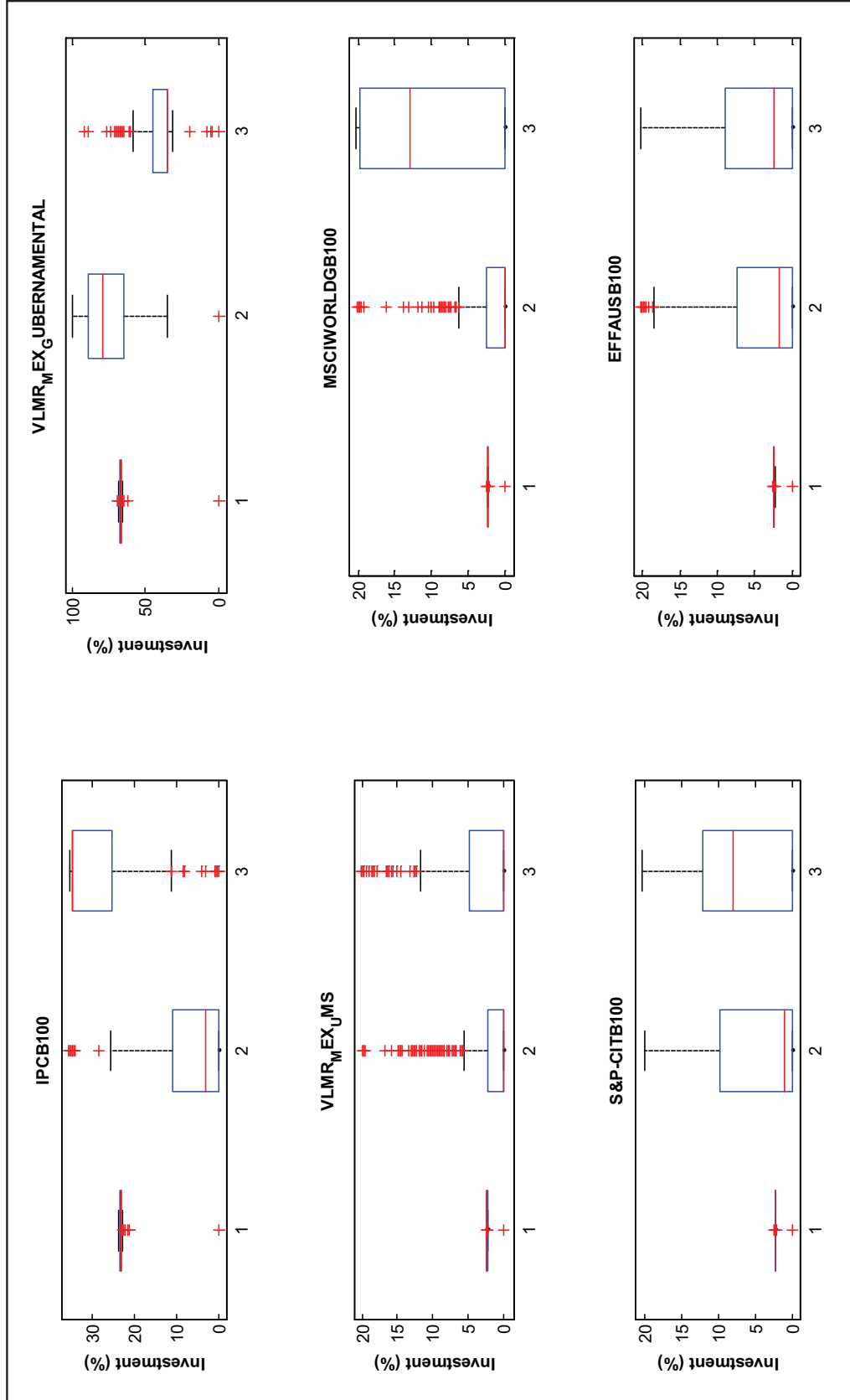


Chart 5. Box plot: Historical investment proportions by market in each portfolio.



Portfolio code (box plot): 1= Target portfolio –passive-, 2= Constant covariance –active, 3= t-Student O-GARCH covariance –active-

suggests the presence of *home bias* in the asset allocation, the O-GARCH matrix handles. this drawback better thanks to an active and a proper management of risky assets during high volatility and correlation clustering periods.

With the results shown in charts four and five, two questions could be posed: Given the historical asset allocation resumed in chart five, does this higher active investment proportions in risky assets explain the better performance in the O-GARCH simulated portfolio? And, do the financial, political, and economic events have an impact on the behavior of the simulated portfolios, leading to a better performance with the use of a t-Student O-GARCH matrix? In order to answer the first question, Chart six presents the historical performance of the six markets in the IPS of Appendix one along with the historical accumulated value of the 7.5% annual target rate (light area).

As noted, the best performers were the Mexican equity, Mexican sovereign bonds, and Mexican treasury markets. If this historical performance is compared with the investment proportions of Chart four and Chart 5, it should be noted that the highest investment proportions in the O-GARCH models are in these three markets. When inspecting Chart four more closely, the performance of the portfolio analysis in the O-GARCH cases suggests a more sensitive asset allocation in the presence of volatility and correlation clustering, i.e. these two portfolios were better diversified during the most uncertain time periods in the financial markets.

The second question “Do the financial, political and economic events have an impact in the behavior of the simulated portfolios, leading to a better performance with the use of a t-Student O-GARCH matrix?” is answered in Chart seven where the historical behavior of the three simulated portfolios is compared with the financial and economic events shown in Chart six. The most notable period depicted in this chart is when the Lehman Brothers’ chapter 11 filing took place. During this time period, the volatility and correlation clustering effect was more observable.²¹ For this reason and because of their statistical properties, the O-GARCH portfolio had a softer behavior than the benchmark and the equally weighted covariance matrix portfolio when the financial crisis was acute.

²¹ It is also when the optimization problem given in (2) leads to the highest concentration in the risk free asset. Please refer to chart four in comparison with chart six to confirm this and to Appendix three for the proof of the presence of volatility clustering in those periods.

Chart 6. Historical performance of the financial markets used in the IPS of table one related to economic, political and financial events.

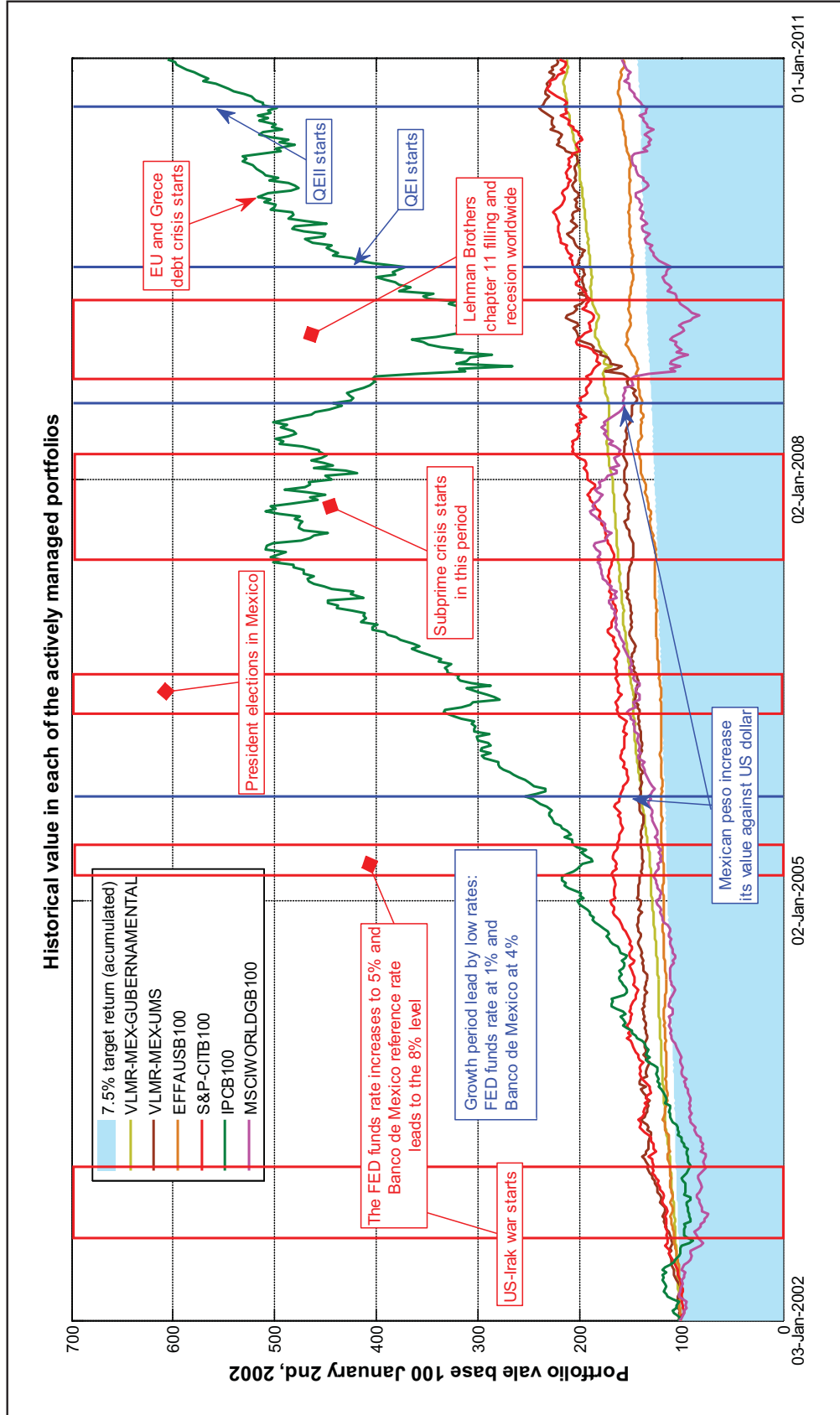


Chart 7. Historical performance of the simulated portfolios related to economic, political and financial events.

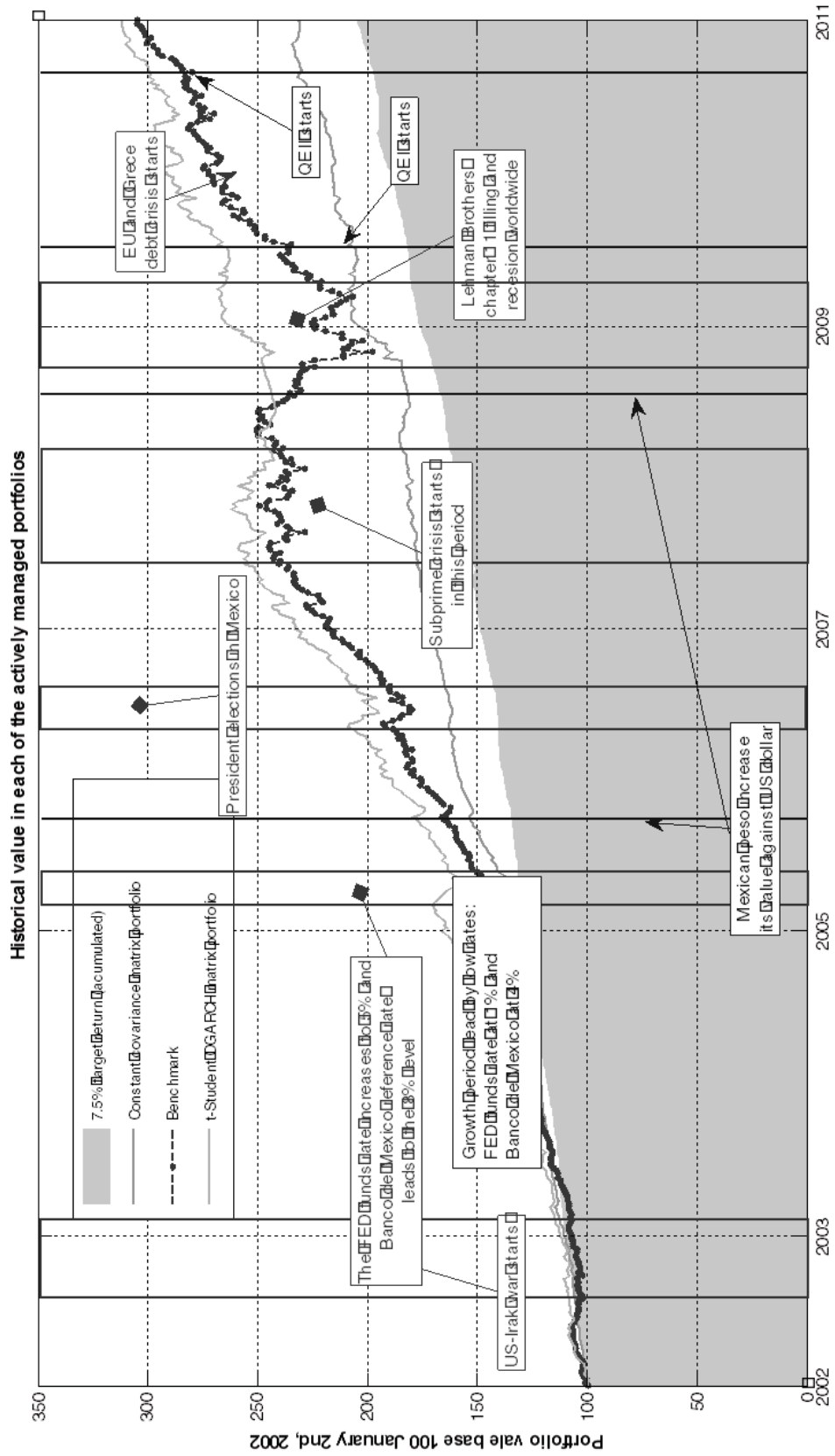


Table 3. ANOVA 1: Test of the historical Sharpe ratios in the four simulated portfolios.

Source	SS	df	MS	F	Prob>F
Columns	43.6680667	3	14.5560222	1.84961698	13.61501%
Error	14393.7718	1829	7.86974947		
Total	14437.4399	1832			

Now that it has been shown that the portfolio management process using the t-Student O-GARCH matrix outperforms a 7.5% annual target return, it is necessary to know the behavior of the risk exposure and turnover relation (financial efficiency) observed, by using this active portfolio management process and covariance matrix to answer the following question: Do we have a higher financial efficiency if an active portfolio management process with a t-Student O-GARCH covariance matrix is used?

To answer this question, the efficiency of the portfolio management process is measured with the historical Sharpe ratio (1966) of each simulated portfolio. If there's either no difference or a lower level of Sharpe ratio (SR) observed in the value of the active O-GARCH portfolio against the other two cases, this portfolio management process is accepted as the best option.

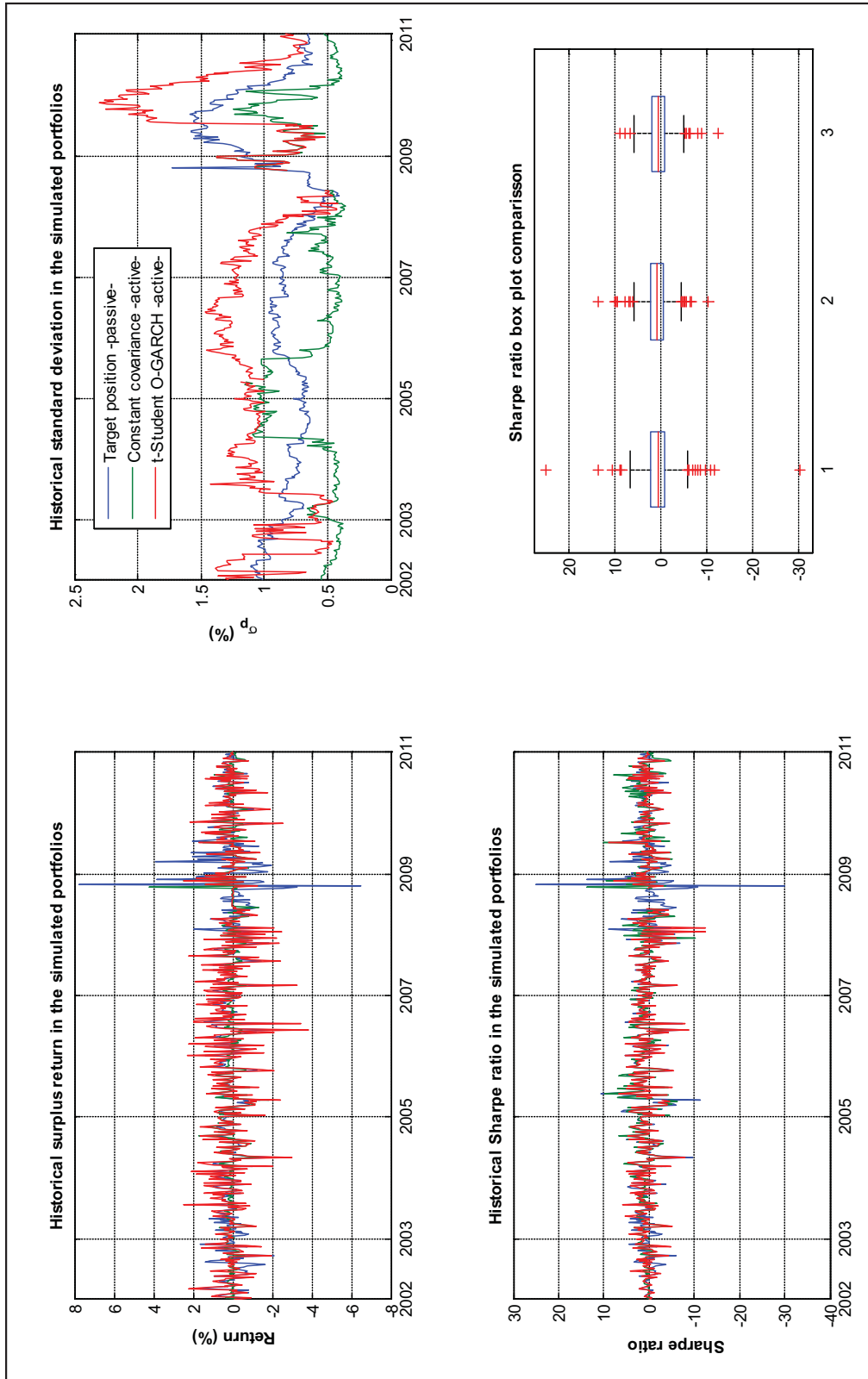
Chart eight presents the historical values observed in each simulated portfolio along with a boxplot comparison. Table three presents the results of a one-way ANOVA test in the historical SR levels, suggesting, along with the results of the boxplot in Chart eight, that the use of a t-Student O-GARCH matrix leads to a better and more stable risk-return trade-off than both the passive and equally weighted covariance matrix portfolios.

4. Concluding remarks.

Given the IPS of Appendix one and from the results observed in the three simulations performed, it is concluded that the *range portfolio rebalancing* discipline with a t-Student O-GARCH matrix in an active portfolio management process is the most suitable for the Technical Reserve of the defined benefit pension fund of interest in this paper (Pensiones Civiles del Estado de Michoacán) and similar ones. This conclusion is supported by the achievement and outperforming of the 7.5% actuarial target return and by a higher turnover than the benchmark (*alpha*), the passively managed and the equally weighted covariance matrix portfolios.

As noted in the results obtained, the use of a t-Student O-GARCH matrix leads to a more suitable asset allocation in the simulated portfolios. This re-

Chart 8. Historical Sharpe ratios for the four simulated portfolios.



Portfolio code (box plot): 1 = Target portfolio –passive-, 2= Constant covariance –active-, 3= t-Student O-GARCH covariance –active-.

mark is confirmed by the fact that it managed, in a better fashion, the investment proportion in the risk-free asset r_f given the presence of volatility and correlation clustering. Also of interest is that the actively managed O-GARCH portfolio was more sensitive to the influence of financial, political, and economic events. This can be observed by using a softer portfolio performance and a more appropriate asset allocation in the risky asset w^* during critical time periods.

As a final remark, it is noted that even though the use of a t-Student O-GARCH Matrix could lead to a higher exposure to risk given the higher return, the observed financial efficiency (risk-return trade off) is higher in this case, supporting the use of this kind of active portfolio management process with this type of covariance matrix.

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Appendix 1

Table 1. Investment Policy Statement of Pensiones Civiles del Estado de Michoacán given its actuarial target return and its future stream of liabilities.

Market	CURRENCY	Index or benchmark	benchmark ticker used in the present paper	Price vendor	Target investment level	investment level allowed	Maximum investment level allowed	Investment level by currency exposure
Mexican government debt market (fixed, float, real and quasi-sovereign debt)	MXN	Valmer México Gubernamental	MEX_GUBERNAMENTAL	Bolsa Mexicana de Valores S.A.B. de C.V. through VALMER	67.061%	51%	100%	MXN exposure
The most traded stocks in the Mexican stock market (IPC index members)	MXN	Índice de Precios y Cotizaciones	IPCB100	Bolsa Mexicana de Valores S.A.B. de C.V.	23.471%	0%	35%	30%-100%
Mexican sovereign debt (UMS)	MXN/1	Valmer UMS	MEX_UMS	Bolsa Mexicana de Valores S.A.B. de C.V. through VALMER	2.367%	0%	20%	USD exposure
United States Treasury bills and bonds markets	USD	EFFA-Bloomberg US treasuries index	EFFAUSB100	Bloomberg Inc. and European Federation of Financial Analysts (EFFA)	2.367%	0%	20%	0%-70%
World treasury bonds markets of the 24 main developed and 24 developing economies (ex US) according to Standard & Poors	USD/2	Standard & Poors - Citigroup international treasury bond index ex-US	S&P-CITB100	S&P Stock Indexes and Citigroup Inc.	2.367%	0%	20%	
World equity markets from the 24 main developed and 24 main developing countries according to MSCI	USD/2	MSCI Global Gross Equity Index USD	MSCIWORLDGB100	MSCI Inc.	2.367%	0%	20%	
Total invested in the index					100.000%			

1/ Even though Mexico has sovereign debt in USD, EUR, JPY and GBP, VALMER values the benchmark in MXN.

2/ Even though the country members quote their assets in their local currencies, the price vendor turn their value to US dollars in order to calculate the benchmark's value. Therefore this benchmark incorporates currency impact.

Appendix 2

Table A.1 The inflows and outflows of the technical reserve of Pensiones Civiles del Estado de Michoacán

Outflows (OF)	Inflows (IF)					
	Date	Amount (MXN)	Date	Amount (MXN)	Date	Amount (MXN)
A monthly \$356.4284 outflow paid the last week of each month. The outflow is the payment of custodial bank services for the assets (ETF's that replicate the six benchmarks) in the managed portfolios.	18/01/2002	\$ 985,516.29	16/07/2004	\$ 641,630.06	02/07/2009	\$ 89,559,398.71
	22/02/2002	\$ 3,285,054.31	10/09/2004	\$ 8,212,635.78	18/12/2009	\$ 66,358,097.13
	22/03/2002	\$ 2,628,043.45	28/01/2005	\$ 82,662,629.71	12/02/2010	\$ 289,453.33
	19/04/2002	\$ 3,285,054.31	08/04/2005	\$ 33,659.65		
	14/06/2002	\$ 2,628,043.45	29/04/2005	\$ 474,292.10		
	09/08/2002	\$ 1,642,527.16	06/05/2005	\$ 3,360.22		
	06/09/2002	\$ 1,642,527.16	19/05/2006	\$ 3,285,054.31		
	20/12/2002	\$ 679,476.88	09/06/2006	\$ 4,927,581.47		
	10/01/2003	\$ 6,570,108.63	24/08/2007	\$ 22,995,380.20		
	02/05/2003	\$ 3,285,054.31	07/09/2007	\$ 8,212,635.78		
	01/08/2003	\$ 4,927,581.47	25/01/2008	\$ 19,710,325.88		
	05/09/2003	\$ 5,256,086.90	28/03/2008	\$ 6,570,108.63		
	19/09/2003	\$ 4,927,581.47	25/04/2008	\$ 6,570,108.63		
	09/01/2004	\$ 6,570,108.63	02/05/2008	\$ 218,473.54		
	07/05/2004	\$ 4,927,581.47	05/06/2009	\$ 163,404,217.91		

Source: Pensiones Civiles del Estado de Michoacán. The real numbers were changed due to confidentiality. This numbers reflect the behavior of the original magnitudes.

Appendix 3

This appendix presents the evidence of the volatility and correlation clustering in the six markets (benchmarks) of the investment policy in Table one. In order to test the presence of the volatility clustering, the Engle (1982) ARCH test was performed in each asset and in each of the weekly dates used in the discrete event simulations. A 95% confidence level is used to test the next hypothesis:

$$H_0 : T \cdot R^2 > X_{95\%,T}^2 \tag{13}$$

Where R^2 is the coefficient of determination of the next auxiliary regression given $\varepsilon_t = r_t - \bar{r}_t$:

$$\varepsilon_t^2 = \alpha + \beta_i \varepsilon_{t-i}^2 \tag{14}$$

This test was performed on each weekly date used for the simulation from January 2, 2002, to December 31, 2010.²² The results are presented in Chart

A.1 and the number of dates with a presence of the ARCH effect is summarized in Table A.2.

As noted in Chart A.2, not all the dates presented an ARCH effect, suggesting that not in all of them an O-GARCH matrix should be used in the MTSL model. In order to accept a general use of GARCH models in all the dates, a Poisson probability function hypothesis test is used with a mean of $\lambda = 23$ and a 95% confidence level given by $\lambda + (95\% \cdot \lambda) = 28.10$. With these parameters, the number of dates that report the presence of the ARCH effect were compared, and if this number was higher than 28.10, the presence of the ARCH effect was accepted for all the dates by assuming that the number of dates is high enough to generalize the presence of this phenomenon in each asset.

The results of these hypotheses tests are presented in the right panels of Chart A.2 and Table A.3. It can be shown that almost all the benchmarks (excepting the US treasuries -EFFAUSB100- that is not conclusive) lead to the acceptance of the ARCH effect for all dates.

Table A.2. ARCH effect test resume

Benchmark	Poisson critical value	Number of dates with ARCH effect	Conclusion
VLMR-MEX-GUBERNAMENTAL	28.10529586	100	This asset has ARCH effect
VLMR-MEX-UMS	28.10529586	55	This asset has ARCH effect
IPCB100	28.10529586	31	This asset has ARCH effect
S&P-CITB100	28.10529586	51	This asset has ARCH effect
EFFAUSB100	28.10529586	28	The test is not conclusive
MSCIWORLDGB100	28.10529586	60	This asset has ARCH effect

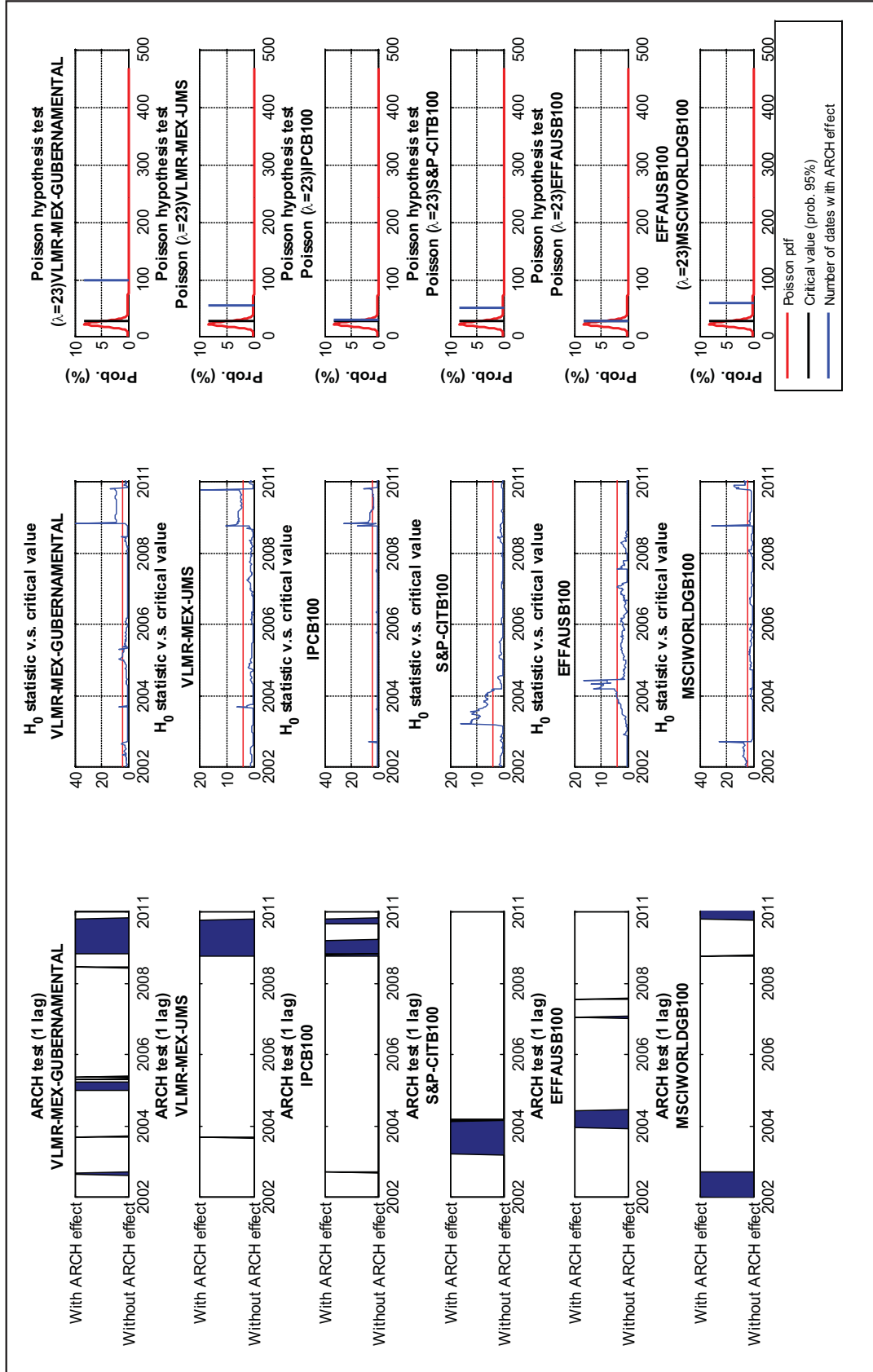
Once the evidence of the ARCH effect in the six benchmarks is presented, it is necessary to demonstrate the usefulness of an O-GARCH covariance matrix by testing the presence of the correlation clustering effect. In order to do so, the return time series \mathbf{r}_i in each asset was divided into two time groups by using the following distance suggested by Chow, Kritzman & Lowry (1999):

$$\varphi_t = (\mathbf{r}_t - \bar{\mathbf{r}}_0)' \cdot \mathbf{C}_0^{-1} (\mathbf{r}_t - \bar{\mathbf{r}}_0) \quad (15)$$

Where \mathbf{r}_t is a 6×1 vector with the observed return at date t in each asset, $\bar{\mathbf{r}}_0$ is a vector with the means of the entire time series \mathbf{r}_t in each asset and \mathbf{C}_0 the covariance matrix for the same data:

²² Using a T=52 return time series length from t to t-51.

Chart A.2. Engle's ARCH test in the six markets used in the asset allocation (benchmarks).



$$C_0 = \left[\left(\left[\mathbf{I} - \frac{1}{n} \cdot \mathbf{1} \cdot \mathbf{1}' \right] \cdot \mathbf{X} \right)' \cdot \left(\left[\mathbf{I} - \frac{1}{n} \cdot \mathbf{1} \cdot \mathbf{1}' \right] \cdot \mathbf{X} \right) \right] \cdot \frac{1}{n}, \mathbf{X} = [\mathbf{r}_1, \dots, \mathbf{r}_i] \quad (16)$$

Each date t or returns vector \mathbf{r}_t from that date was included in the “unusual dates” set Θ by following the rule:

$$\mathbf{r}_t \in \Theta \Leftrightarrow \rho_t > X_{95\%, \nu}^2 \quad (17)$$

Where ν are the degrees of freedom related to the number of assets included in the covariance matrix C_0 . Once Θ and Θ^C are defined with (16), two correlation matrixes were calculated for the usual and unusual date sets and the correlation of Θ^C was compared with Θ , leading to the results shown in Table A.3.

As noted, the correlation observed in “unusual times” increased in eight of 15 pairs of assets (or markets), suggesting the presence of correlation clustering in turbulent or unusual times. This can also be observed in the difference of the effective correlation (determinant) value observed in both matrixes.

Table A.3 Correlation matrix comparisson in "unusual" and "usual" dates.

"Unusual dates" correlation matrix.						
Asset type	VLMR_MEX_UMS	EFFAUSB100	SP500TRB100	MSCIWORLDGB100	VLMR_MEX_GUBERNAMENTAL	IPCB100
VLMR_MEX_UMS	1					
EFFAUSB100	0.408287007	1				
SP500TRB100	-0.340421318	-0.417051245	1			
MSCIWORLDGB100	-0.352979265	-0.354964665	0.91810741	1		
VLMR_MEX_GUBERNAMENTAL	-0.04527618	0.196305779	0.099137752	0.110340901	1	
IPCB100	-0.010496348	0.05721501	-0.052602401	0.008871604	-0.019101146	1
Effective correlation (Determinant)	0.089648924					
"Usual dates" correlation matrix.						
Asset type	VLMR_MEX_UMS	EFFAUSB100	SP500TRB100	MSCIWORLDGB100	VLMR_MEX_GUBERNAMENTAL	IPCB100
VLMR_MEX_UMS	1					
EFFAUSB100	-0.117425925	1				
SP500TRB100	-0.369673719	-0.286097744	1			
MSCIWORLDGB100	-0.416630382	-0.255187292	0.9557821	1		
VLMR_MEX_GUBERNAMENTAL	0.255163266	-0.057908116	0.481861901	0.49041005	1	
IPCB100	-0.143888126	0.365550275	-0.08956764	-0.049046676	-0.151424226	1
Effective correlation (Determinant)	0.023188107					
Correlation level difference between "unusual" and "usual" dates						
Asset type	VLMR_MEX_UMS	EFFAUSB100	SP500TRB100	MSCIWORLDGB100	VLMR_MEX_GUBERNAMENTAL	IPCB100
VLMR_MEX_UMS	0					
EFFAUSB100	0.525712932	0				
SP500TRB100	0.0292524	-0.130953501	0			
MSCIWORLDGB100	0.063651118	-0.099777373	-0.03767469	0		
VLMR_MEX_GUBERNAMENTAL	-0.300439446	0.254213895	-0.382724149	-0.380069149	0	
IPCB100	0.133391778	-0.308335266	0.036965239	0.05791828	0.132323079	0
Determinant difrence	0.066460817					