Full cooperation to solve environmental problems using financial markets

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Resumen

El presente estudio muestra que el camino más eficiente para mejorar el medio ambiente en cualquier área y en cualquier dimensión es con el uso de una versión *suave* de la metodología de Agente-Principal mediante la emisión de certificados de mejora que abarquen entidades grandes y así estimular la transferencia de tecnologías. Comparamos soluciones óptimas en dos casos: acción conjunta y acción tipo fusión. *Fusión* significa que un agente puede hacer mejoras en el dominio de otro. Analizamos modelos específicos y relativamente sencillos en los que se pueden obtener resultados explícitos. Estos modelos los usamos como modelos de referencia, puesto que su justificación práctica, incluyendo la calibración de los parámetros, representa el mayor defecto en aplicaciones de fenómenos ambientales. Aunque la metodología podría ser aplicada, en teoría a muchos problemas ambientales, el presente estudio está enfocado básicamente a problemas de contaminación. Trabajaremos con procesos de difusión introduciendo el factor de cooperación.

Keywords Cooperation between agents, pollution, diffusion processes. *Mathematical Subject Classification*: 65K10, 69J60, 62P12.

Abstract

This study shows that the most efficient way to improve the natural environment, is by using a "soft" version of the Principal-Agent methodology by means of the emission of improvement certificates that embrace large entities and therefore stimulate the transference of technologies. We compare two cases of optimal solutions, collusive optima and fusion optima. "Fusion" means that one agent-owner of a certificate- can make improvements in another agent's domain, (which we'll call "land"). We analyze specific and relatively simple models for which explicit or almost explicit solutions can be reached. We use these models as reference models only because their practical justification, including the calibration of parameters, is its major flaw when applied to environmental phenomena. Although the methodology could, in theory, be applied to several environmental problems, this study deals basically with pollution. We work with difusion processes, introducing the cooperation factor.

Introduction

There have been many attempts to use financial markets to combat or diminish environmental deterioration. Among the most significant are the following:

1. Emission trading (known as cap-and-trade) related to permits to pollute.

2. Economic evaluation based on "willingness to pay" known as full "cost-benefit" analysis.

3. Establishment of property of rights, i.e. the privatization of nature.

4. Valuing the environment through contingent valuation. (Hanemann, 1994).

However, none of the above mentioned approaches have rendered the desired effects.

1. The main deficiency of the "permits to pollute" approach is that it does not stimulate any cooperation. If someone finds a new method to capture carbon (just to give one example), there is no reason to believe that he or she would share this invention with others instead of profiting by selling permits. Besides, it leads to a wild market with strong governmental intervention, for example, assigning initial quota, this is called the *grandparent effect*.

2. Full economical analysis needs very precise models but natural phenomena are far too complex and depend on too many processes to be fully understood or measured. M. Sagoff rightly stressed that "(…) the immense effort economists have invested over decades in trying to measure the benefits of environmental resources and services has resulted and can result only in confusion", (Sagoff, 2004).

3. Establishing property rights that require institutional arrangements and procedures that are difficult to accomplish, should not be proposed as a solution to the "tragedy of the commons". Natural resources are hard to privatize. Even dealing with deforestation (this being a problem that mostly affects developing countries), the attempt to set property rights encounters increasing social problems rather hard to solve.

4. The Contingent valuation method (CVM) is used to estimate economic values of all kinds of ecosystems and environmental goods by asking how much one would be willing to pay for a specific good. Unfortunately the answers were closely related to the educational level of people involved and the kind of questions asked.

Although the comprehensive conservation of the biological diversity requires a strategy that goes beyond cost-benefit analysis –the monetary valuation can play a supportive role in environmental policy, but its multiple practical and normative problems have to be considered when using such a method, above all in developing countries where people are too poor to think about environmental degradation. Philip E. Graves wrote: "To the extent that we value public goods, we also realize that getting extra income to buy them will accomplish nothing", (Graves, 2003). It was A. Fitzsimmons who, in his controversial book *Defending illusions*, pointed out the possibility of creating markets on environmental topics. He assumed that the Wetland Protection Certificated could be bought and sold, and that a market may be established by the US Congress (1999).

In this article we assume that an environmental fund has already been created and could be used by two parties. To describe these entities we'll use the word *land*. By the same token, we propose a "conditional carrot" approach, based on a free market made out of certificates of improvements. Trading exclusively good certificates cannot endow environmental goods with market values.

Presently, we analyze different aspects of cooperation in three mathematical models:

1. Elementary deterministic model.

2. Squared Bessel processes with linear improvements only. We show how state dependent agents' actions can reduce the cooperation

factor needed to make fusion worthy. We also explain how to analyze and value certificates in this particular setting.

3. General processes with external factor independent of agents' actions, (Cadenillas *et al*, 2004).

We use the *soft* version of the Principal –Agent methodology, being Nature the Principal and the Agents would be the individuals or institutions (most of them with nil participation). Certificates can be sold or given out free of charge (for instance in the case of poor countries or natural environmental damage). To make it easier, we analyze the case of two *lands* only, but the method could be extended to include many *lands* in a simple way.

In all cases, we consider that the payment would increase when pollution levels diminish. Principal problem-optimality of certificates, needs precise estimation of social costs of pollution, and these estimations seem to be *hardly accessible*. That´s why we call our approach *soft*. While we use the easiest example of equal *lands*, no problem arises on how much each land ought to contribute to the funds, or how to distribute possible gains. The corresponding fund should be large enough to create positive total net gain. In the opposite case, there is a possibility of debt, or falling in a kind of default, which should be analyzed with more sophisticated techniques-like put options. Unused funds could be used to finance other projects. A different approach, with the use of the Principal-Agent method has been considered by D'Amato and Franckx, (D'Amato, Franckx, 2003). They wrote: "We have considered there the regulation of a (private or public) agent by an EPA (Environmental Protection Agency). This EPA is constrained to basing its incentive scheme (both rewards and punishments) on environmental performance, and allocate funds to alternative projects with environmental benefits. The private agent can allocate its effort to environmental protection or to its core task". While we consider only environmental improvements, we go further in cooperation topics. At the same time, our approach does not need precise specification of parameters, as the above quoted study requires.

The approach presented here, pretends to open the path toward practical solutions to prevent environmental destruction.

1. Very simple deterministic model.

To explain what we understand by *fusion* let us start with two simple examples of identical lands.

These examples with general parameters have been studied by Ray, who compared competitive Nash and collusive equilibria, (Ray, 2001). The mismatch between the equilibria above mentioned disappears automatically with emissions of certificates of improvement.

The real advantage of fusion could be appreciated in more complicated stochastic models.

Let each "land" emit pollution level and loss function are given by

$$A(1-X_i)^2+\frac{X_i^2}{2},$$

where the second term represents its social cost that is unknown, and set just for illustration purpose, and the first term is the cost for abatements.

In this example we assume that pollution in one land doesn't affect the counterpart. Now optimal $X_i^*=\frac{A}{1+A}$ and the total level of pollution is $2X_i^*$.

Consider fusion and assume that joint loss function is given by:

$$\frac{X_1^2}{2}+\frac{X_2^2}{2}+\frac{(2-Y)^2}{2}A_1,$$

where $Y=X_1+X_2$.

If we assume that $A_1=\frac{1}{B}A$, then for $B>2$ we get

$$\frac{4A_1}{1+2A_1}<\frac{2A}{1+A}.$$

We will call *B cooperation factor* (for unequal lands can be smaller than two).

On the other hand, when we consider neighboring lands (pollution in one affects the other in a straightforward way), and social loss function $\frac{(X_1+X_2)^2}{2}$, then $X_i^* = \frac{A}{2+A}$, and total level of pollution equals $2X_i^*$.

In the case of fusion the total loss being

$$A_1 \frac{(2-Y)^2}{2} + \frac{Y^2}{2}$$

and once again the total level of pollution is smaller when $A_1 = \frac{1}{B}A,\ B > 2$.

Therefore, emissions of certificates of improvement in the form

$$S - \frac{(X_1 + X_2)^2}{2}$$

will lead directly to fusion if $B > 2$.

2. Cooperation in the case of $BESQ_\beta^\delta$ processes.

In this part we would like to show how to value different certificates and explain how to do it for state dependent actions (linear with fixed parameters) the cooperation factor needed to make fusion worthy would be smaller than 2. (Similar conclusions can be obtained in more general models).

To begin with, we would like to formulate some basic facts about these processes and comment the modeling in this setting.

$BESQ_\beta^\delta$ process is defined by

$$dX(t) = 2\sqrt{X(t)}dW(t) + (2\beta X(t) + \delta)dt, \qquad X(0) > 0.$$

(We will set $X(0) = 1$).

We assume that $\delta \geq 0$, but at this moment do not specify the sign of β. If $\beta = 0$ then $X(t)$ becomes squared Bessel process of dimension δ . For more properties of $BESQ_{\beta}^{\delta}$ see (Revuz & Yor, 1999). $\beta < 0$ $BESQ_{\beta}^{\delta}$ is known in finance as CIR model (from Cox, Ingersoll & Ross) for instantaneous interest rates.

$BESQ_{\beta}^{\delta}$ processes can be obtained as a heavy traffic approximation of corresponding Piecewise Deterministic Markov Processes with some environmental motivation.

Pollution levels are increasing with time, but at some random moments actions are taken to combat it. See (Szatzschneider & Jeanblanc, 2002) for details of application to forest issues.

In our example we will compare agents actions for certificates of the form

$$S - \int_0^1 \big(X(s) + Y(s)\big)^2 \, ds.$$

For $X(s), Y(s)$ independent $BESQ_{\beta}^{\delta}$ processes, $X(s) + Y(s)$ become $BESQ_{\beta}^{2\delta}$ process.

There are well known formulas for $E\big(X(s)\big)$, $Var\big(X(s)\big)$, and $E\left(e^{\sigma \int_0^1 X(s)ds}\right)$ for $\beta < 0$ and sufficiently small positive σ, see appendix for calculations of the expectation for negative σ.

Therefore, one can design many reasonable certificates with explicit valuations.

Now, if agents actions are limited to making δ smaller, then the cooperation factor B will still be two, as before. But if an agent can make δ and β smaller (we call this action linear improvements), then the B needed will be smaller than two. Exact calculations clearly depend on chosen parameters.

Coming back to our example:

$$dX(t) = 2\sqrt{X(t)}dW_1(t) + (2\beta X(t) + \delta)dt,$$

$$dY(t) = 2\sqrt{Y(t)}dW_2(t) + (2\beta Y(t) + \delta)dt.$$

Assume that an agent can make improvements in his (her) land, changing $\delta \to \delta - \frac{1}{2}\delta_1$, with total cost of improvements (for both lands) $2\int_0^1 \left(\frac{1}{2}\delta_1\right)^2 dt$ and in the case of fusion $\frac{1}{B}\int_0^1 \delta_1^2 dt$, arriving in both cases at the same pollution levels. To make pollution level smaller, one has to cut off costs, so must be greater than two.

Assume now that one agent can make improvements changing $\beta \to \beta - \beta_1$, and $\delta \to \delta - \frac{1}{2}\delta_1, \beta_1 > 0, \delta - \delta_1 > 0$,

Let $E\big(X(t)\big) = f(t)$ and $E\big(X^2(t)\big) = g(t)$.

The cost of improvements is individually $\int_0^1 \left(\beta_1 X(s) + \frac{\delta_1}{2}\right)^2 ds$ and the same is true for $Y(s)$, while in the case of fusion the total cost is

$$\frac{1}{B}\int_0^1 \left[\beta_1\big(X(s) + Y(s)\big) + \delta\right]^2 ds.$$

Therefore, the joint cost in the collusive case is:

$$2\beta_1^2 g(t) + 2\beta\delta f(t) + \frac{\delta_1^2}{2},$$

and in the fusion:

$$\frac{1}{B}(2\beta_1^2 g(t) + 2\beta_1^2 f^2(t) + 4\delta\beta_1 f(t) + \delta_1^2)$$

and clearly even for $B < 2$ the cost of improvements would be smaller.

The exact calculation of will be performed in the next section for a different model.

3. The case of external factor.

In this case we work with different lands and borrow the general idea from (Cadenillas *et al*, 2004).

We will deal with a more general case. However we solve exclusively the Agent Problem.

The pollution level is modeled as

$$dS(t) = \delta u(t)dt + \square dt + \circledcirc dW(t),$$

$S(0) > 0$, $\square$, and $\circledcirc$ is here fixed adapted to Brownian filtration processes, independently of the agent's action $u(t)$.[1]

Assume also that there will be a unique strong solution to this equation for our choice of $u(t)$ that will be specified soon and $S(t) > 0,\ 0 \leq t \leq 1$.

This model will produce a very simple optimal solution for in the case of certificates:

$$F - \int_0^1 S(u)du.$$

Let us rewrite $\int_0^1 S(u)du$ as

$$S - S(0) - \int_0^1 (1-t)dS(t) = \tilde{S} - \int_0^1 \pi_t dS(t).$$

The Agent's problem is to maximize

$$E\left[U\left(\tilde{S} - \int_0^1 (1-t)dS(t)\right) - \int_0^1 G\big(u(s)\big)ds\right],$$

being U some unspecified increasing utility function of the Agent.

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1 Los símbolos $\square$ y $\circledcirc$ fueron elegidos por los autores para subrayar la robustez del modelo

$$\text{Let } P_t = \int_0^1 \pi_u \, dS(u), \ 0 \leq t \leq 1.$$

Now

$$d\left(P_t - \int_0^t G\big(U(s)\big)ds\right) = \pi_t \square dt + \pi_t \circledcirc dW(t) + [\delta \pi_t u_t - G(u_t)]dt.$$

Because neither $\square$, $\circledcirc$ nor π_t depends on u_t, and assuming a quadratic cost of improvements $G_t = \frac{u_t^2}{2}$, we obtain that the optimal agent's action is $\hat{u}_t = \delta \pi_t$ and is independent of the utility U.

We would like to compare fusion versus collusive optimality.

Let

$$dS_1(t) = \delta_1(t-1) + \circledcirc_1 dW_1(t) - \square_1 dt$$

$$dS_2(t) = \delta_2(t-1) + \circledcirc_2 dW_2(t) - \square_2 dt,$$

W_1, W_2 being independent.

Joint expectation (assuming that stochastic integrals are true martingales) is:

$$E\left(\int_0^1 (\square_1 + \square_2)dt\right) - \frac{1}{2}(\delta_1^2 + \delta_2^2).$$

while fusion solution for $E(S_1 + S_2)$ would give $E\left(\int_0^1 (\square_1 + \square_2)dt\right) - \frac{\delta^2}{2}$ so fusion provides better results meanwhile $\delta > \sqrt{\delta_1^2 + \delta_2^2}$.

Conclusions

The choice of certificates could be the subject of separate studies and matched in some sense to the social costs of pollution. However, it is very difficult to estimate them correctly. Therefore, for the first two applications the approach can be any of the proposed. We leave to the market the exact costs of improvements.

It is frequent in quantitative finance that the design of financial products anticipates their valuation. We propose first to apply, and later to discuss and analyze the performance of the certificates of improvements. For one agent's problem see (Szatzschneider & Kwiatkowska, 2010). More detailed descripcion of fusion can be found in (Szatzschneider & Kwiatkowska, 2009).

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Appendix

Let

$$dY(t) = 2\sqrt{Y(t)}dW_2(t) + (2\beta Y(t) + \delta)dt,$$

set $Y(0) = 1$ (to simplify).

We will formulate here the result that will be useful to prove Theorem 2.

Theorem 1

Let $\sigma > 0$

$$E\left(e^{-\sigma\int_0^1 Y(t)dt}\right) = \varphi(1)^{\frac{1}{2}} exp\left[\frac{1}{2}\left(\varphi'^{(0)} - \beta(\delta+1)\right)\right] \quad (1)$$

where φ is the solution of

$$\frac{\varphi''(s)}{\varphi(s)} = \beta^2 + 2\sigma, \quad s\epsilon(0,1), \quad \varphi(0 = 1) \quad (2)$$

$$\frac{\varphi'(1)}{\varphi(1)} = \beta \text{ (left hand derivative)} \quad (3)$$

This theorem is a particular case of one presented in (Szatzschneider, 2002) and also appears in (Musiela & Rutkowski, 2009).

Clearly it is very easy to calculate φ:

$$\varphi(s) = Ae^{cs} + (1 - A)e^{-cs}$$

with $c = \sqrt{2\sigma + \beta^2}$ and is calculated from (3).

This result has been obtained with the use of exponential martingales, Girsanov, and integration by parts.

Theorem 2

Assume $\beta < 0$. The formula (1) is valid for $E\left(e^{\sigma \int_0^1 Y(t)dt}\right)$, $\sigma > 0$, with the following modifications:

Case 1: $\beta^2 - 2\sigma > 0$

In this case $\varphi(s)$ has the same form as in theorem 1, with $c = \sqrt{2\sigma + \beta^2}$.

Case 2: $\beta^2 - 2\sigma = 0$

Now $\varphi(s) = \frac{\beta}{1-\beta} s + 1.$

Case 3: $0 < 2\sigma - \beta^2 < \frac{\pi^2}{4}$

Set $\gamma = \sqrt{2\sigma - \beta^2}$,

$$\varphi(1) = \frac{\gamma}{\gamma \cos\gamma - \beta \sin\gamma}$$

$$\varphi(0) = \gamma \left(\frac{\gamma \sin\gamma + \beta \cos\gamma}{(\gamma - \beta)\cos\gamma}\right).$$

Proof:

Apply the same martingale method as in theorem 1.

Case 1: The proof is done exactly as in theorem 1.

Cases 2 and 3: The bounded solution of (2) and (3) exists in (0,1), and also for Case 3, $\cos\gamma s + A \sin\gamma s$ with A calculated as before from $\frac{\varphi'(1)}{\varphi(1)} = \beta < 0.$